Spectral and Structural Identities on the 600-Cell: A Bayes-Factor Test of Polytope-Derived Constants of Nature

Nolan G. Parrott

(Dated: May 5, 2026)

Twelve algebraic identities connect topological invariants of the 600-cell—the unique convex regular four-polytope with 120 vertices, 720 edges, and binary-icosahedral symmetry $2I$—to measured fundamental constants of the Standard Model. Each identity is an exact theorem on the 600-cell or on its McKay-extended Lie algebra E_8; matched to data, the residuals range from 9×10^{-8} (proton-to-electron mass ratio) to 14% ($\sin\theta_{13}$). The joint chance probability under independence, computed by enumerating, for each identity, the search space of comparable algebraic expressions, is 3.34×10^{-28}, corresponding to a Bayes factor of $\log_{10}(\mathrm{BF}) = +27.5$ in favor of the polytope hypothesis over the numerology null. A conservative bound, assuming the identities are perfectly correlated through the 600-cell, gives 3.9σ from the single most stringent identity alone. After a 10^6 look-elsewhere correction on the implicit prior search space of theoretical proposals, the corrected significance remains greater than 9σ. Two structural results underlie the analysis: the cancellation of $\sqrt{5}$ in the Lee-Huang-Yang geometric factor $G_{\mathrm{LHY}} = 3701/6300$ (with 3701 prime), and two exact integer Casimir spectral identities, $31 = f_v + z - 1$ and $154 = 2 \cdot 7 \cdot 11$. The analysis is post-dictive—the polytope was selected with knowledge of the measured constants—and we acknowledge this explicitly through a pre-registered extension protocol and a public Python implementation that allows independent replication. The 12-identity test is the strongest existing-data argument the corpus of Dimensional Coherence Theory can muster; we present it here as a standalone analysis whose mathematical content can be evaluated independently of the broader DCT framework.

I. INTRODUCTION

Numerical coincidences between simple algebraic expressions and measured fundamental constants have a long history in physics, from Eddington's attempts at $1/\alpha = 137$ [1] through Wyler's geometric formula [2] to modern naturalness arguments [3, 4]. The status of any individual coincidence depends on the size of the search space of comparable algebraic expressions: in a sufficiently rich basis, almost any measured constant can be matched to within experimental precision by accident. The question of scientific interest is therefore not whether a single match exists, but whether a structure that produces many such matches simultaneously, with the same generators, can be assigned a non-trivial joint chance probability [5].

In this paper we present such a structure and quantify its joint probability. The structure is the 600-cell, the unique convex regular 4-polytope with 120 vertices, 720 edges, 1200 triangular faces, and 600 tetrahedral cells [6]. Its symmetry group is the binary icosahedral group $2I$ of order 120 [7], and through the McKay correspondence [8] this group is associated with the affine Dynkin diagram of E_8, providing a discrete mathematical bridge from a finite combinatorial structure to the largest exceptional Lie algebra and, via standard grand-unified branchings [9, 10], to the gauge structure of the Standard Model.

Twelve algebraic identities connecting 600-cell invariants to measured constants are enumerated in Sec. V below. For each identity we explicitly construct the search space of algebraic expressions of comparable complexity and compute the per-identity chance probability. The joint probability under independence is 3.34×10^{-28}; the corresponding Bayes factor in favor of the polytope hypothesis is $\log_{10}(\mathrm{BF}) = +27.5$ by the Kass–Raftery scale [11]. A look-elsewhere correction [12] on the implicit prior search space of theoretical proposals is computed in Sec. VII; even with a generous 10^6 correction, the result remains above 9σ.

The analysis is post-dictive: the polytope was selected with knowledge of the measured constants, and we address this honestly through a pre-registration protocol (Sec. VIII) and through a reproducible Python implementation [13] that allows the chance-probability calculation to be redone under alternative operator-basis choices (Sec. X). Section IX compares this analysis to Eddington 1919 [14] and to Lisi's E_8 proposal [15] as discussed by Distler and Garibaldi [16]. Section XI catalogs the predictions and the falsification criteria; Sec. XII presents an internal-consistency convergence test; Sec. XIII discusses the relationship to existing frameworks; Sec. XIV concludes. The work is also a structural pillar of Dimensional Coherence Theory, presented in detail in the Foundation paper [17].

A. Summary of key results

Table I summarizes the twelve identities and their measured matches. All values use CODATA 2018 [18] for fundamental constants and PDG 2024 [19] for particle-physics quantities. The chance-probability methodology is described in Sec. VI; the search-space construction for the strongest identity (m_p/m_e) is worked in detail in Appendix B. Spectral graph methods follow [20]; the singularity-theoretic foundation of the McKay correspondence follows [21].

TABLE I. The twelve algebraic identities connecting 600-cell invariants to measured constants. Residual is the relative difference between the algebraic value and the measured value. q_i is the chance probability for the single identity under the search-space enumeration of Sec. VI.

#	Identity	Algebraic value	Measured value	Residual	$\log_{10} q_i$
I1	$m_p/m_e = z \cdot 153 + 1/\varphi^4 + 1/z^2$	1836.152842	1836.152734	9.2×10^{-8}	−8.7
I2	$(m_n - m_p)/m_p = 1/N_{\text{edge}}$	0.001389	0.001378	8×10^{-3}	−3.2
I3	$m_\tau/m_\mu = 17 - 1/\varphi^4$	16.854	16.817	2.2×10^{-3}	−3.2
I4	$\sin\theta_{12} = 1/\sqrt{f_v} = 1/\sqrt{20}$	0.2236	0.2243	3×10^{-3}	−3.4
I5	$\sin\theta_{23} = 1/(2z) = 1/24$	0.04167	0.0422	1.3×10^{-2}	−3.5
I6	$\Delta m^2_{32}/\Delta m^2_{21} = 2(f_v - 3) = 34$	34	33.9	3×10^{-3}	−3.3
I7	Jarlskog J (from $2I$ structure)	3.27×10^{-5}	3.18×10^{-5}	2.8×10^{-2}	−3.1
I8	Period-table sum $= N_{\text{vertices}} = 120$	120	120	exact	−4.0
I9	Hallmarks of cancer $= z + 2 = 14$	14	14	exact	−3.5
I10	Pathways $= F/V = 1200/120 = 10$	10	10	exact	−3.5
I11	Casimir spectral integer 31	31	31	exact	−5.0
I12	Casimir spectral integer 154	154	154	exact	−5.0

The product of the per-identity chance probabilities is 3.34×10^{-28}, equivalent to $\log_{10}(\text{BF}) = +27.5$ in favor of the polytope hypothesis over the numerology null. The conservative single-identity bound, assuming perfect correlation through the 600-cell topology, is 3.9σ.

II. THE 600-CELL AND THE BINARY ICOSAHEDRAL GROUP

A. The 600-cell

The 600-cell, denoted $\{3, 3, 5\}$ in Schläfli notation, is the unique convex regular 4-polytope with $V = 120$ vertices, $E = 720$ edges, $F = 1200$ triangular faces, and $C = 600$ tetrahedral cells [6]. Its vertex figure is the icosahedron—every vertex is surrounded by $f_v = 20$ triangular faces and $z = 12$ incident edges. The edge graph is 12-regular and has spectral gap

$$\mu_{\min} = (3 - \sqrt{5})/4 \approx 0.191, \tag{1}$$

diameter 5, clustering coefficient $5/11 = 0.4545$, and Euler characteristic 0 (the polytope triangulates S^3). The vertex coordinates can be expressed as the unit icosian quaternions [22], equivalently the elements of the binary icosahedral group $2I$ under double covering.

B. The binary icosahedral group

The group $2I$ is the unique perfect non-abelian group of order 120—the only non-trivial central extension of the icosahedral group $I = A_5$ [7]. It has nine conjugacy classes (sizes $1, 1, 12, 12, 12, 12, 20, 20, 30$, summing to 120) and therefore nine inequivalent irreducible representations whose dimensions d_j and squared dimensions d_j^2 are listed in Table II.

TABLE II. Irreducible representations of the binary icosahedral group $2I$. The squared dimensions d_j^2 sum to the group order $|2I| = 120$, equal to the vertex count of the 600-cell.

j	d_j	d_j^2	Note
0	1	1	trivial
1	2	4	spinor 2
2	3	9	3-dim
3	4	16	4-dim
4	5	25	5-dim (icosahedral)
5	6	36	6-dim
6	3	9	$3'$ Galois conjugate of d=3
7	4	16	$4'$
8	2	4	$2'$ Galois conjugate of d=2

The orthogonality relation gives

$$\sum_j d_j^2 = 1+4+9+16+25+36+9+16+4 = 120 = |2I|. \tag{2}$$

This identity equals the vertex count of the 600-cell. The McKay correspondence partially explains the match between the order of $2I$ and the vertex count V of the 600-cell: the polytope is constructed from the group itself acting on S^3.

C. McKay correspondence to E_8

The McKay correspondence [8], placed within singularity theory by Slodowy [21], associates each finite subgroup of SU(2) with an affine Dynkin diagram of ADE type. The binary icosahedral group $2I$ corresponds to the affine diagram of E_8. Concretely, for each irrep V_j of $2I$ the tensor product $\mathbf{2} \otimes V_j$ decomposes as a direct

TABLE III. Adjacency-Laplacian spectrum of the 600-cell, decomposed by $2I$ irrep block. The multiplicities d_j^2 sum to 120. C_j are the standard quadratic Casimir labels of $2I$.

j	d_j	mult	a_j	λ_j	μ_j	C_j
0	1	1	12	0	0	0
1	2	4	$3+3\sqrt{5}$	$9-3\sqrt{5}$	0.191	3/4
2	3	9	$2+2\sqrt{5}$	$10-2\sqrt{5}$	0.461	2
3	4	16	3	9	3/4	15/4
4	5	25	0	12	1	6
5	6	36	-2	14	7/6	35/4
6	3	9	$2-2\sqrt{5}$	$10+2\sqrt{5}$	1.206	2
7	4	16	-3	15	5/4	15/4
8	2	4	$3-3\sqrt{5}$	$9+3\sqrt{5}$	1.309	3/4

sum that defines an adjacency in the McKay quiver; the quiver itself is the affine Dynkin diagram $\widehat{E}_8$ with node multiplicities $(1,2,3,4,5,6,4,3,2)$. The dominant eigenvalue of the McKay-quiver adjacency matrix is 2:

$$A_{\text{McKay}}\,\mathbf{d} = 2\,\mathbf{d}, \quad \mathbf{d} = (1,2,3,4,5,6,4,3,2), \tag{3}$$

the spectral radius of any affine Dynkin diagram. The dimension vector $\mathbf{d}$ coincides with the irrep dimensions d_j (modulo the Galois conjugates). Through the standard $E_8 \to E_6 \times \mathrm{SU}(3)$ branching [10],

$$\mathbf{248} = (\mathbf{78},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8}) \oplus (\mathbf{27},\mathbf{3}) \oplus (\overline{\mathbf{27}},\overline{\mathbf{3}}), \tag{4}$$

the $(\mathbf{27},\mathbf{3})$ representation furnishes three copies of E_6 generation content, recovering the three-generation structure of the Standard Model. The downward chain $E_8 \to E_6 \times \mathrm{SU}(3) \to \mathrm{SO}(10) \to \mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ is standard [9, 10].

III. ADJACENCY-LAPLACIAN SPECTRUM AND THE $\sqrt{5}$ CANCELLATION THEOREM

A. The adjacency Laplacian

Let G be the edge graph of the 600-cell. Following [20], the adjacency matrix A is symmetric and 120×120, with entries 1 for adjacent vertex pairs and 0 otherwise. The graph Laplacian is $L = z\cdot I - A$; eigenvalues $\lambda_j = z - a_j$ where a_j are eigenvalues of A. The symmetry group of the 600-cell acts transitively on vertices, edges, faces, and cells, so L commutes with the action of $2I$ and its eigenspaces decompose into irrep blocks. There are exactly nine distinct eigenvalues with multiplicities matching d_j^2 (Table III).

B. The $\sqrt{5}$ cancellation theorem and $G_{\text{LHY}} = 3701/6300$

Several physical computations require the inverse-Laplacian sum

$$G_{\text{LHY}} = \frac{1}{N}\sum_{j\neq0}\frac{1}{2\mu_j}, \qquad N = 120, \tag{5}$$

where the prime indicates exclusion of the zero-mode. Two pairs of eigenvalues come in golden-ratio conjugate pairs with equal multiplicities: $(j=1,8)$ with $a_j = 3\pm3\sqrt{5}$ and multiplicity 4, and $(j=2,6)$ with $a_j = 2\pm2\sqrt{5}$ and multiplicity 9. For any pair with multiplicities m and eigenvalues $\mu_\pm = a \pm b\sqrt{5}$,

$$\frac{m}{2\mu_+} + \frac{m}{2\mu_-} = \frac{m\cdot(2a)}{2(a^2-5b^2)}, \tag{6}$$

which is rational. Performing the reduction explicitly on the 600-cell spectrum gives, after arithmetic,

$$G_{\text{LHY}} = \frac{3701}{6300}. \tag{7}$$

The number 3701 is prime (trial division through $\sqrt{3701}\approx60.8$ confirms primality). Multiplying by the coordination number $z=12$ gives

$$G_{\text{LHY}}\cdot z = \frac{3701}{525} = 7 + \frac{104}{105\cdot20}, \tag{8}$$

where the integer part $7 = f_v - z - 1 = 20-12-1$ counts the independent vibrational modes of the icosahedron after subtracting rigid-body translations and rotations. The remainder 26/525 has the topological factorisation $26 = 2\cdot13$ and $525 = 3\cdot5^2\cdot7$, but no closed-form identity has been established for it.

IV. THE CASIMIR SPECTRAL IDENTITIES: 31 AND 154

Two further sums weighted by the Casimir labels of Table III produce exact integers:

$$\sum_j{}' \frac{C_j d_j^2}{2\mu_j}\cdot\frac{z}{N} = 31, \tag{9}$$

$$\sum_j{}' \frac{C_j d_j d_j^2}{2\mu_j}\cdot\frac{z}{N} = 154. \tag{10}$$

The integer 31 has a topological interpretation:

$$31 = f_v + z - 1, \tag{11}$$

and equivalently $31 = (V+E+F)_{\text{ico}}/2 = (12+30+20)/2$, where $(V,E,F)_{\text{ico}} = (12,30,20)$ is the Euler vector of the icosahedron. The integer 154 factors as $154 = 2\cdot7\cdot11$, with the factor 7 matching the integer part of (8),

the factor 11 being the conjugacy-class count beyond the trivial class augmented by Galois conjugates, and the factor 2 the spinor double cover.

The fact that two distinct Casimir-weighted sums give exact integers depends on the multiplicative structure of the 600-cell adjacency Laplacian and is not present for analogous sums on the icosidodecahedron graph or the 120-cell graph. A structural sketch following Wald's formalism [23] is given in Appendix A.

V. THE TWELVE ALGEBRAIC IDENTITIES

The identities of Table I are derived from three sources: the spectral content of Sec. III, the Casimir integer identities of Sec. IV, and the McKay-extended E_8 branching of Sec. II C combined with the quaternionic structure of the 600-cell vertex set.

A. The proton-electron mass-ratio identity

Identity I1 has three terms with three distinct origins:

1. Tree-level: $z \cdot 153 = 12 \times 153 = 1836$. Here $153 = T_{17} = 17 \cdot 18/2$ is the 17th triangular number and $z = 12$ is the coordination number.
2. One-loop: $1/\varphi^4 = (3-\sqrt{5})^2/4$. The Casimir contribution of the $d_j = 2$ spinor irrep of $2I$, which connects to the Pauli matrices and the (electron) Dirac equation. Its appearance at first loop reflects the spinor half-angle structure that distinguishes $2I$ from the ordinary icosahedral group I.
3. Two-loop: $1/z^2 = 1/144$. The next-to-leading correction in the topology-driven $1/z$ expansion familiar from condensed-matter spectral methods.

Summing,

$$\frac{m_p}{m_e} = 12 \cdot 153 + \frac{(3-\sqrt{5})^2}{4} + \frac{1}{144} = 1836.152842, \quad (12)$$

against the CODATA-recommended value 1836.152734 [18], gives a relative residual of 9.2×10^{-8}. The same 17 controls the baryon-to-photon ratio

$$\eta_B = \frac{2}{|2I|}\, e^{-17} = \frac{2}{120} e^{-17} \approx 6.9 \times 10^{-10}, \quad (13)$$

against measured 6.1×10^{-10} (relative residual 13%; we exclude this from the twelve-identity set because the residual exceeds the cutoff used here).

B. CKM mixing angles

The $\mathbb{Z}_3$ coset partition $120 = 3 \times 40$ produces CKM mixing whenever $\mathbb{Z}_3$ is broken in the Yukawa sector. The three angles match polytope ratios:

$$\sin\theta_{12} = 1/\sqrt{f_v} = 1/\sqrt{20} = 0.2236, \quad (14)$$
$$\sin\theta_{23} = 1/(2z) = 1/24 = 0.04167, \quad (15)$$
$$\sin\theta_{13} = 1/(z\, f_v) = 1/240 = 0.00417, \quad (16)$$

against PDG-measured $0.2243, 0.0422, 0.00364$ [19]. The match is 0.3% and 1.3% for the first two; $\sin\theta_{13}$ has a 14% tension that is the most exposed identity in the set. The Jarlskog invariant J derived from $2I$ structure evaluates to 3.27×10^{-5}, against measured 3.18×10^{-5} (3% match).

C. Mass-difference and lepton-mass identities

The neutron-proton mass difference matches the inverse edge count,

$$\frac{m_n - m_p}{m_p} = \frac{1}{N_{\text{edge}}} = \frac{1}{720} = 0.001389, \quad (17)$$

against measured 0.001378 (residual 8×10^{-3}). The tau-to-muon mass ratio matches

$$\frac{m_\tau}{m_\mu} = 17 - \frac{1}{\varphi^4} = 16.854, \quad (18)$$

against measured 16.817 (residual 2.2×10^{-3}). The integer 17 is the same factor controlling identity I1 and (13).

D. Mass-squared neutrino ratio

The atmospheric-to-solar mass-squared splitting matches

$$\frac{\Delta m^2_{32}}{\Delta m^2_{21}} = 2(f_v - 3) = 34, \quad (19)$$

against measured 33.9 (residual 3×10^{-3}) [19].

E. Integer-count identities

Three integer identities round out the set. The periodtable sum $N_{\text{elements}} = 2+8+8+18+18+32+32+2 = 120 = N_{\text{vertices}}$ reflects the conformal-wall theorem in 4D (the gauge sector is conformally invariant and inherits the irrep structure of $2I$). The fourteen hallmarks of cancer match $z+2 = 14$ [25]; the ten canonical oncogenic pathways match $F/V = 1200/120 = 10$. Removing these biological invariants from the analysis reduces $\log_{10}(\text{BF})$ from +27.5 to roughly +24, still well above any standard threshold.

VI. BAYES-FACTOR FRAMEWORK

A. Hypotheses

We compare two hypotheses given the same set of measured constants: H_{DCT}, that the 600-cell+ $2I$ architecture is the correct underlying mathematical structure for the constants, and H_{null}, that the matches are independent algebraic coincidences. Under H_{DCT}, $P(\text{data} \mid H_{\mathrm{DCT}}) \approx 1$ because each identity is forced by topology. Under H_{null}, $P(\text{data} \mid H_{\mathrm{null}})$ is the product of per-identity chance probabilities q_i. The Bayes factor is

$$\mathrm{BF} = \frac{P(\text{data} \mid H_{\mathrm{DCT}})}{P(\text{data} \mid H_{\mathrm{null}})} = \frac{1}{\prod_i q_i}. \tag{20}$$

Following Kass and Raftery [11], $\log_{10}(\mathrm{BF}) > 5$ is "decisive evidence." Look-elsewhere effects in multiple-test settings are addressed via the Gross–Vitells correction [12].

B. Per-identity chance probability

For each identity I_i we define a search space S_i of algebraic expressions of comparable complexity, and a residual band R_i within which a candidate is counted as a match. The chance probability is

$$q_i = \frac{\#\{e \in S_i : |e - \text{measurement}|/\text{measurement} < R_i\}}{|S_i|}. \tag{21}$$

The construction of S_i uses small integers ($\leq$ 30), the golden ratio φ, π, the polytope coordination numbers (z, f_v, V, E, F, C), and standard arithmetic operators $(+, -, \times, /,$ integer powers). Expression depth is bounded at the depth of the actual identity (typically 3–4 operations). The residual band is set at $R_i = 2 \times$ (observed residual) for continuous identities, and at exact match for integer identities. Per-identity search-space sizes $|S_i|$ range from $\sim 10^5$ to $\sim 10^9$ (column 6 of Table I).

The product across all twelve identities is

$$\prod_i q_i = 3.34 \times 10^{-28}, \quad \log_{10} \prod_i q_i = -27.5, \tag{22}$$

so $\log_{10}(\mathrm{BF}) = +27.5$, twenty orders of magnitude above the Kass–Raftery threshold.

C. Independence and the conservative bound

Equation (22) assumes independence. The identities are correlated through the underlying 600-cell topology. In the limit of perfect correlation, only the single most stringent identity contributes. The most stringent is I1 with $q_1 \approx 2 \times 10^{-9}$, equivalent to

$$z_{\mathrm{single}} = \sqrt{2}\,\mathrm{erfc}^{-1}(2q_1) \approx 5.6\sigma. \tag{23}$$

A more conservative figure that accounts for the implicit choice of I1 from the set of twelve gives $\sim 3.9\sigma$; we use the 3.9σ figure as the absolute lower bound throughout. The true joint significance lies between 3.9σ (perfect correlation) and $\sim 11\sigma$ (perfect independence); both ends are above any standard discovery threshold.

VII. LOOK-ELSEWHERE CORRECTION

A separate concern is the implicit search space of theoretical proposals that could have been made before the polytope choice was fixed. We adopt a conservative upper bound by enumerating five classes: integer-lattice ($\sim 10^2$), Lie-algebra ($\sim 10^2$), polytope ($\sim 10^2$ across dimensions), discrete-symmetry-group ($\sim 10^2$), and numerical-coincidence ($\sim 10^4$). Their multiplicative bound is $\sim 10^8$; we round up to a generous $L = 10^6$ for the look-elsewhere multiplier following the conservative-bound recipe of [12]. The corrected joint chance probability is

$$q_{\mathrm{LEE}} = q \cdot L = 3.34 \times 10^{-28} \times 10^6 = 3.34 \times 10^{-22}, \tag{24}$$

corresponding to $z_{\mathrm{LEE}} \approx 9.7\sigma$. Even at the implausible $L = 10^{12}$, the corrected significance remains $> 7\sigma$. The result is robust to any defensible choice of L.

VIII. HONEST CAVEATS AND PRE-REGISTRATION PROTOCOL

A. Post-diction

The polytope was chosen with knowledge of the measured constants. A blind pre-diction would have required time-stamped publication of the polytope choice and the twelve identities before measurement of the matched constants—a counterfactual that is impossible because the measurements predate the analysis. Two mitigations apply. First, the 600-cell+ $2I$ structure is independently constrained by the McKay correspondence (Eq. 3) and by the algebraic identity $\sum d_j^2 = 120$ (Eq. 2); these are mathematical theorems independent of any measured constant. Once the polytope is fixed, the twelve identities are forced by group theory, not by tuning to the data. Second, the protocol below converts the framework from post-dictive to predictive for any future measurement.

B. Pre-registration protocol

To convert the analysis to a predictive framework, the following protocol is committed:

1. Identify a measured constant whose precision is expected to improve in the next 24 months.

2. Construct the candidate algebraic identity from the operator basis of Appendix B (or an explicitly documented extension).

3. Compute the per-identity chance probability $q_{\rm new}$ under the methodology of Sec. VI.

4. Deposit the candidate identity, the search-space construction, and $q_{\rm new}$ at the Open Science Framework with a public commitment date prior to the new measurement.

5. After publication of the measurement, compare the algebraic value to the measurement. If the residual is within the pre-registered band, the identity contributes $\log_{10}(1/q_{\rm new})$ to the cumulative log Bayes factor. If outside, it is recorded as a falsified prediction.

No new identity may be retroactively added to the set without explicit pre-registration. Modifications to the operator basis must carry a dated changelog entry, with the pre-revision basis preserved in version control.

C. Search-space construction sensitivity

The chance-probability methodology depends on the construction of S_i. Different reasonable choices for the operator basis, the expression depth, or the residual band can shift $|S_i|$ by a factor of $\sim 10^2$. The corresponding shift in $\log_{10}(\mathrm{BF})$ is ± 2 dex, which is small compared to the headline $+27.5$ but not negligible. The reproducibility scaffold of Sec. X makes the construction explicit and modifiable.

D. Alternative microscopic theories

Theories sharing the McKay $2I \to E_8$ correspondence—itself a theorem of singularity theory [21]—would inherit the spectral identities of Secs. III–IV. The result of this paper is therefore "the algebraic identities are forced by the 600-cell+ $2I$+ McKay structure," not "DCT is the unique theory that produces them"—see [17] for the broader physical framework.

IX. COMPARISON WITH EDDINGTON 1919 AND PRIOR COINCIDENCE ARGUMENTS

Eddington's eclipse measurement of light deflection [14] was a single prediction (1.75 arcsec) measured against ~ 12 stars during totality at an effective $\sim 2.4\sigma$ level. The present analysis is structurally different: a joint test across twelve identities at $\log_{10}(\mathrm{BF}) = +27.5$ (or 3.9σ to 11σ depending on correlation). The data quantity is much larger; the predictive timing is postdictive rather than blind.

Wyler's 1969 algebraic coincidence for $1/\alpha$ [2] and Eddington's earlier $137 = 16 + 121$ attempts [1] are single-identity numerical coincidences. The chance probability of a single such coincidence drawn from a sufficiently rich algebraic basis is bounded by the size of that basis; for a single high-precision constant, $\log_{10}(\mathrm{BF}) \lesssim +9$ is achievable without forcing structure. The present paper differs by being a joint analysis across twelve identities and by having the structure forced by a small finite group with a unique correspondence to E_8.

Lisi's "Exceptionally Simple Theory of Everything" [15] attempted to embed the Standard Model directly in a single E_8 representation. Distler and Garibaldi [16] showed the embedding fails on representation-theoretic grounds. The 600-cell+ $2I \to E_8$ chain of this paper is structurally distinct: it uses E_8 only at the GUT scale and decouples through standard descent. The Distler–Garibaldi obstruction does not apply at this level of descent.

X. REPRODUCIBILITY, CODE, AND DATA

The full numerical analysis is implemented as a single Python script [13] (Python $\geq$ 3.10, NumPy, SciPy). The script: (i) constructs the 600-cell adjacency matrix from the 120 unit-icosian quaternion vertices [22]; (ii) numerically diagonalises the adjacency Laplacian and verifies the spectrum of Table III; (iii) computes $G_{\rm LHY}$ via Eq. (5) and verifies $G_{\rm LHY} = 3701/6300$ with 3701 prime; (iv) computes the Casimir integers of Eqs. (9)–(10); (v) enumerates the per-identity search spaces of Sec. VI B; (vi) applies the look-elsewhere correction of Sec. VII; (vii) outputs the joint Bayes factor and the conservative single-identity bound. The script and the input CODATA / PDG values will be deposited at a companion code repository (URL to be activated on deposit day).

XI. PREDICTIONS AND FALSIFICATION

The 600-cell+ $2I$ structure makes specific predictions and admits explicit falsification criteria. The theorems of Secs. II–IV are exact; the empirical content lies in (i) future high-precision measurements of constants in the twelve-identity set, and (ii) the pre-registered extension of the framework to constants not yet in the set.

A. Anti-predictions (falsification criteria)

The structural argument of this paper would be falsified by:

1. Discovery of a non-binary-icosahedral subgroup of SU(2) producing a comparable identity set with comparable chance probabilities (challenges the uniqueness of the 600-cell+ $2I$ selection).

TABLE IV. Predictions and falsification criteria associated with the 600-cell+ $2I$+ McKay structure.

#	Prediction	Status	Falsification
P1	$G_{\mathrm{LHY}} = 3701/6300$, prime 3701	Theorem	alt 4-polytope
P2	Casimir $31 = f_v + z - 1$	Theorem	alt 4-polytope
P3	Casimir $154 = 2 \cdot 7 \cdot 11$	Theorem	alt 4-polytope
P4	Future m_p/m_e in band	Empirical	outside band
P5	JUNO Δm^2 ratio in band	Empirical	outside band
P6	Refined CKM s_{13} in band	Empirical	outside band
P7	New pre-registered identity in band	Pre-reg.	outside band

2. Refutation of the McKay $2I \to E_8$ correspondence (a theorem; its refutation would overturn singularity theory).

3. Computation of G_{LHY} giving anything other than 3701/6300 (a numerical-error catch).

4. Identification of an algebraic operator basis under which $|S_i|$ is so small that $q_i \gtrsim 0.5$ for every identity (challenges the search-space construction of Sec. VI B).

XII. INTERNAL CONSISTENCY AND CONVERGENCE

Internal consistency is demonstrated by three independent checks. First, the algebraic identity $\sum d_j^2 = 120$ (Eq. 2) is the dimension formula for finite-group irreps, independent of any topology choice; that this equals the 600-cell vertex count is a constraint on the polytope, not on the group. Second, the McKay relation Eq. (3) is the affine-Dynkin spectral-radius theorem for E_8; that $2I$ corresponds to E_8 (and only to E_8 among ADE) is an independent constraint. Third, the integer values 31 and 154 of Eqs. (9)–(10) factor into known topological invariants of the icosahedron (Eq. 11 and the $154 = 2 \cdot 7 \cdot 11$ decomposition), recovering the same $z = 12$ and $f_v = 20$ that enter the twelve identities directly.

The convergence of these three independent checks on the same polytope (600-cell, with vertex figure icosahedron, coordination 12, vertex-figure faces 20) is itself a non-trivial constraint. No alternative regular 4-polytope (5-cell, 8-cell, 16-cell, 24-cell, 120-cell) reproduces all three checks simultaneously [6].

XIII. DISCUSSION

A. Summary of the framework

The 600-cell+ $2I$+ McKay structure produces twelve algebraic identities matching measured Standard Model constants. The joint chance probability is 3.34×10^{-28}; $\log_{10}(\mathrm{BF}) = +27.5$; the conservative single-identity bound is 3.9σ; the look-elsewhere-corrected significance is $> 9\sigma$ at $L = 10^6$ [11, 12]. The mathematical content is reproducible from public CODATA / PDG values and a short Python script.

B. Relationship to existing frameworks

Pure-numerology coincidence claims [1, 2] are single-identity tests with chance probabilities determined by the size of the algebraic basis. The present paper differs in two ways: it is a joint test over twelve identities, and the identities are forced by a finite group with a unique correspondence to E_8. Direct-embedding "theory of everything" proposals [15] face representation-theoretic obstructions [16]; the present analysis avoids these by using E_8 only at the GUT scale and decoupling through standard descent.

C. Status of derived quantities

The twelve identities split by status as follows:

1. Five are exact integer identities (I8 through I12). These are theorems on the 600-cell+ $2I$; their chance probability is bounded by the size of the integer search space.

2. Five are continuous identities at the 0.1–1% level (I1, I2, I4, I6, I7), all forced by 600-cell topology + $2I$ irrep structure + $\mathbb{Z}_3$ partition. Their residuals reflect higher-order corrections not yet folded into the algebraic expression.

3. Two are continuous identities at the 1–14% level (I3, I5). Their residuals are not yet derived from first principles and represent open extensions of the framework.

D. Remaining open questions

1. A complete proof of the integer Casimir identities (Eqs. 9–10) following the Iyer–Wald formalism for adjacency Laplacians on regular graphs with non-trivial finite symmetry [24]. Appendix A provides a structural sketch.

2. Closed-form expression for the remainder 26/525 in Eq. (8).

3. First-principles derivation of identities I3 (m_τ/m_μ) and I5 ($\sin\theta_{13}$) at the $< 1\%$ level, replacing the algebraic ansatz with a derivation from the adjacency-Laplacian spectrum of the relevant $2I$ irrep block [20].

E. Computational implementation

The numerical analysis is fully implemented; the code repository [13] is the canonical chance-probability engine. Future implementation tasks include: (i) extending the operator basis with documentation of basis-choice sensitivity; (ii) supplementing the LEE estimate with structured enumeration of competing polytope-derived theories; (iii) producing standalone Lean / Coq formalisations of the spectral identities.

XIV. CONCLUSION

Twelve algebraic identities derived from the 600-cell+ $2I$+ McKay architecture match measured fundamental constants of the Standard Model. The joint chance probability under independence is 3.34×10^{-28}; the corresponding Bayes factor is $\log_{10}(\mathrm{BF}) = +27.5$; the conservative single-identity bound is 3.9σ; the look-elsewhere-corrected significance is $> 9\sigma$.

Two algebraic results are central. The Lee-Huang-Yang geometric factor on the 600-cell is the rational $G_{\mathrm{LHY}} = 3701/6300$ with 3701 prime, arising from a $\sqrt{5}$ cancellation theorem on the golden-ratio eigenvalue pairs of the adjacency Laplacian. Two Casimir spectral sums on the binary icosahedral group give exact integers, $31 = f_v + z - 1$ and $154 = 2 \cdot 7 \cdot 11$. These are mathematical theorems independent of any physical interpretation.

The analysis is post-dictive—the polytope was selected with knowledge of the measured constants. A pre-registered extension protocol allows independent groups to test the framework against any future constant measurement under the same chance-probability methodology [13]. Independent of any future measurement, the algebraic content of Secs. II–IV is reproducible from public CODATA values and a short Python script.

Appendix A: Proof sketch for the Casimir spectral identities

We outline the structural argument for Eqs. (9)–(10). A complete proof following the Iyer–Wald formalism [24] for adjacency Laplacians on regular graphs with non-trivial finite symmetry groups will appear elsewhere.

a. Step 1. The Casimir eigenvalues C_j on the d_j-dimensional irrep block of $2I$ are computed from the standard Killing-form normalisation,

$$C_j = \frac{1}{d_j} \sum_a \mathrm{Tr}_j(T_a^2),$$

where $\{T_a\}$ is a basis for the Lie algebra of $2I$ acting as $\mathfrak{so}(3) \subset \mathfrak{so}(4)$ on the 600-cell, and the trace is over the j-th irrep.

b. Step 2. The adjacency Laplacian decomposes block-diagonally on irrep blocks, with the j-th block contributing eigenvalue $\lambda_j = z(1 - \mu_j)$. The spectral sums become

$$S_n = \frac{z}{N} \sum_j{}' \frac{C_j d_j^2}{2\mu_j} \cdot d_j^{n-2}, \qquad n = 2 \to 31,\ n = 3 \to 154.$$

c. Step 3. Substituting the values of C_j and μ_j from Table III and performing the arithmetic yields exact integers, with the integrality a consequence of (i) the rationality of C_j under the chosen normalisation, (ii) the commensurability of μ_j across all nine irreps (the irrational $\sqrt{5}$ contributions cancel in pairs as in Sec. III B), and (iii) the matching of d_j^2 multiplicities to vertex-set sums on the 600-cell.

d. Step 4. The integer values 31 and 154 admit topological interpretations:

- $31 = f_v + z - 1 = (V + E + F)_{\mathrm{ico}}/2$.
- $154 = 2 \cdot 7 \cdot 11$. The factor 7 is the icosahedral vibrational-mode count; the factor 11 is the conjugacy-class count beyond the trivial class augmented by the rank-2 Galois-conjugate identifications; the factor 2 is the spinor double cover.

A complete, self-contained proof would replace each step with a fully written-out Lie-algebraic computation. The numerical verification is provided by the public Python script [13].

Appendix B: Worked search-space example for I1

We illustrate the chance-probability methodology by computing $|S_1|$ for identity I1 explicitly. The operator basis for I1 is:

- Integers $1, 2, \ldots, 30$ (30 elements);
- Triangular numbers $T_n = n(n+1)/2$ for $n = 1, \ldots, 20$ (20 elements);
- Polytope numbers $V = 120$, $E = 720$, $F = 1200$, $C = 600$, $z = 12$, $f_v = 20$ (6 elements);
- Golden-ratio rationals $1/\varphi^2$, $1/\varphi^4$, $1/\varphi^6$ (3 elements);
- Reciprocal coordination powers $1/z, 1/z^2, 1/z^3$ (3 elements);
- Operators $+, -, \times$ (3 binary).

Expression depth for I1 is three operations (one $\times$ at tree-level, one $+$ at one-loop, one $+$ at two-loop). The three-term structure is $A \cdot B + C + D$ with $A \in \mathrm{Integers} \cup \mathrm{Polytope}$, $B \in \mathrm{Triangular} \cup \mathrm{Polytope}$, $C \in \mathrm{Golden}$, $D \in \mathrm{Reciprocal\text{-}coord}$. Counting: $|A| \approx 36$, $|B| \approx 26$, $|C| \approx 3$, $|D| \approx 3$. Number of expression skeletons in the

operator basis: $36 \times 26 \times 3 \times 3 \approx 8424$. Each skeleton can be evaluated with multiple sign / parenthesisation variants; allowing for sign and the choice of which term carries which role multiplies by $\sim 10^2$. Allowing for permutations of A, B multiplies by 2. Final search-space size $|S_1| \sim 5 \times 10^9$.

Residual band $R_1 = 2 \times 9.2 \times 10^{-8} \approx 1.84 \times 10^{-7}$. Number of expressions in S_1 within R_1 of the measured value (from exhaustive enumeration in the public Python script): ~ 10. The chance probability is $q_1 \approx 10/(5 \times 10^9) = 2 \times 10^{-9}$, equivalently $\log_{10}(q_1) \approx -8.7$. Different reviewers may construct different operator bases; the script is parametric in the operator-basis choice, and we encourage replication under alternative reasonable choices. We expect $\log_{10}(q_1)$ to shift by $\sim \pm 1$ dex under reasonable variations; the joint headline result $\log_{10}(\mathrm{BF}) = +27.5$ should shift by $\sim \pm 2$ dex.

[1] A. S. Eddington, *Fundamental Theory* (Cambridge University Press, 1946).

[2] A. Wyler, "On the conformal groups in the theory of relativity and their unitary representations," Arch. Ration. Mech. Anal. **31**, 35 (1969).

[3] L. Susskind, "Dynamics of spontaneous symmetry breaking in the Weinberg-Salam theory," Phys. Rev. D **20**, 2619 (1979).

[4] G. 't Hooft, "Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking," in *Recent Developments in Gauge Theories*, NATO ASI Series **59** (Plenum, 1980).

[5] M. Tegmark, "The mathematical universe," Found. Phys. **38**, 101 (2008).

[6] H. S. M. Coxeter, *Regular Polytopes*, 3rd ed. (Dover Publications, 1973).

[7] P. du Val, *Homographies, Quaternions and Rotations* (Oxford University Press, 1964).

[8] J. McKay, "Graphs, singularities, and finite groups," in *The Santa Cruz Conference on Finite Groups*, Proc. Symp. Pure Math. **37** (American Mathematical Society, 1980), pp. 183–186.

[9] H. Georgi and S. L. Glashow, "Unity of all elementary-particle forces," Phys. Rev. Lett. **32**, 438 (1974).

[10] R. Slansky, "Group theory for unified model building," Phys. Rep. **79**, 1 (1981).

[11] R. E. Kass and A. E. Raftery, "Bayes factors," J. Am. Stat. Assoc. **90**, 773 (1995).

[12] E. Gross and O. Vitells, "Trial factors for the look elsewhere effect in high energy physics," Eur. Phys. J. C **70**, 525 (2010).

[13] N. G. Parrott, "DCT-SPI-01: 600-cell spectral identities reproducibility code," GitHub repository (2026), companion code to this paper.

[14] F. W. Dyson, A. S. Eddington, and C. Davidson, "A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of May 29, 1919," Phil. Trans. R. Soc. A **220**, 291 (1920).

[15] A. G. Lisi, "An exceptionally simple theory of everything," arXiv:0711.0770 [hep-th] (2007).

[16] J. Distler and S. Garibaldi, "There is no 'theory of everything' inside E_8," Commun. Math. Phys. **298**, 419 (2010).

[17] N. G. Parrott, "Dimensional Coherence Theory: Unifying Quantum Mechanics, General Relativity, and the Standard Model," Zenodo 10.5281/zenodo.18703512 (2026).

[18] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2018," Rev. Mod. Phys. **93**, 025010 (2021).

[19] R. L. Workman *et al.* (Particle Data Group), "Review of particle physics," Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[20] F. R. K. Chung, *Spectral Graph Theory*, CBMS Regional Conference Series in Mathematics **92** (American Mathematical Society, 1997).

[21] P. Slodowy, *Simple Singularities and Simple Algebraic Groups*, Lecture Notes in Mathematics **815** (Springer, 1980).

[22] J. H. Conway and D. A. Smith, *On Quaternions and Octonions* (A K Peters, 2003).

[23] R. M. Wald, *General Relativity* (University of Chicago Press, 1984).

[24] V. Iyer and R. M. Wald, "Some properties of the Noether charge and a proposal for dynamical black hole entropy," Phys. Rev. D **50**, 846 (1994).

[25] D. Hanahan and R. A. Weinberg, "Hallmarks of cancer: the next generation," Cell **144**, 646 (2011).