


Standard-Model Gauge Structure from McKay $2I \rightarrow E_8$ in Dimensional Coherence Theory

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The Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ and its three-generation fermion content are derived from the McKay correspondence between the binary icosahedral group $2I$ — the symmetry of the 600-cell — and the affine Lie algebra E_8 . Through the standard branching $E_8 \rightarrow E_6 \times SU(3)_{\text{family}} \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$, three fermion generations are forced by the $(\mathbf{27}, \mathbf{3})$ representation of $E_6 \times SU(3)$, and the \mathbb{Z}_3 coset partition $120 = 3 \times 40$ of the 600-cell vertex set produces CKM mixing in the Yukawa sector. The proton-to-electron mass ratio is derived as $m_p/m_e = z \cdot 153 + 1/\varphi^4 + 1/z^2 = 1836.152842$, matching CODATA 1836.152734 to 9.2×10^{-8} . The CP-violating phase is $\delta_{\text{CP}} = \pi/3 = 60^\circ$ from the spinorial half-angle lift of $C_3 \subset SO(3)_L \subset SO(4) \subset H_4$ to $\text{Spin}(4)$, with residual 5.5° (3.6σ) bracketed by Standard Model running at 7° – 12° . The Jarlskog CP-violation invariant $J = 3.27 \times 10^{-5}$ derived from the same \mathbb{Z}_3 structure of the $2I$ vertex partition matches the PDG measurement $J = 3.18 \times 10^{-5}$ at 3% (0.6σ); this is one of the corpus’s three genuine pre-dictive wins (alongside the SHOES H_0 match at 0.02σ and the JWST τ_{reion} shift at 1.2σ [1]). The post-hoc closures $\sin \theta_{13} = (1 - 1/\varphi^4)/240$ (1.69%) and $1/\alpha = \varphi^5 \cdot 4\pi \cdot (1 - \mu^2/2)$ (0.16%) are explicitly labelled as algebraic ansatzes, not first-principles derivations, and are the targets of Programs A and B in the corpus master roadmap [1]. The \mathbb{Z}_3 cosmic-string tension $G\mu = 2.7 \times 10^{-6}$ vs. Planck bound 1.1×10^{-7} is documented as the structural open question of the Standard-Model sector. The paper presents the SM-from-topology content of the corpus master scorecard [1] as a standalone particle-physics test paper.

I. INTRODUCTION

The Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ and its three-generation fermion content are the most successful empirical inputs of high-energy physics, yet remain unexplained at the level of fundamental theory. Grand-unified extensions [3, 4] embed the SM gauge group in a larger simple group such as $SU(5)$, $SO(10)$, or E_6 , but the three-generation structure is typically inserted by hand or attributed to anthropic / phenomenological selection.

The McKay correspondence [5, 6] provides a structural alternative. Each finite subgroup of $SU(2)$ corresponds to an affine Dynkin diagram of ADE type, with the binary icosahedral group $2I$ — the symmetry of the 600-cell — corresponding to the affine \widehat{E}_8 . Through the standard $E_8 \rightarrow E_6 \times SU(3)$ branching, the $(\mathbf{27}, \mathbf{3})$ representation furnishes three copies of E_6 generation content, and the further descent $E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ recovers the SM gauge group. The three generations are not chosen — they are a consequence of the McKay correspondence between $2I$ and E_8 .

Dimensional Coherence Theory (DCT) [1] adopts this McKay-extended structure as the topological origin of the Standard Model gauge sector. The Brans–Dicke amplitude P on the 600-cell graph carries the gauge-sector phase θ as a Goldstone mode of the underlying $2I$ symmetry, and the Standard Model gauge fields are identified with the Goldstone modes of the McKay-extended $\widehat{E}_8 \rightarrow E_6 \times SU(3) \rightarrow SO(10) \rightarrow SU(5) \rightarrow \text{SM}$ chain.

This paper presents the SM-sector predictions of DCT: the gauge group, the three-generation structure, the proton-to-electron mass ratio at 9.2×10^{-8} precision, the

CKM mixing angles, the neutrino mass-squared splitting ratio, the CP-violating phase $\delta_{\text{CP}} = \pi/3$ (with the corpus history of the retracted $2\pi/3$ explicitly addressed), and the post-hoc closures of the fine-structure constant and $\sin \theta_{13}$. We acknowledge the \mathbb{Z}_3 cosmic-string tension as the structural open question of this sector.

A. Summary of key results

The two non-trivial structural matches are m_p/m_e at 9.2×10^{-8} and the McKay-derived gauge group plus three-generation count. The CKM mixing angles match to 0.3–1.3% for $\sin \theta_{12}$ and $\sin \theta_{23}$; $\sin \theta_{13}$ is post-hoc closure at 1.69%. The CP-violating phase $\delta_{\text{CP}} = \pi/3$ from the \mathbb{Z}_3 spinorial branch matches at 3.6σ , bracketed by Standard Model running.

II. THE MCKAY CORRESPONDENCE AND THE GAUGE GROUP

A. Binary icosahedral group

The 600-cell, the unique convex regular 4-polytope with $V = 120$ vertices and $E = 720$ edges, has symmetry group H_4 of order 14,400. Its rotation subgroup is $H_4^+ \sim SO(4)/\mathbb{Z}_2$, and the lift to $SU(2) \times SU(2)$ contains as a diagonal subgroup the binary icosahedral group $2I$ of order 120. The $2I$ group has nine conjugacy classes (sizes 1, 1, 12, 12, 12, 12, 20, 20, 30, summing to 120) and therefore nine inequivalent irreducible representations.

The McKay correspondence [5] associates $2I$ with the affine Dynkin diagram \widehat{E}_8 . Concretely, for each irrep V_j

TABLE I. DCT Standard-Model sector predictions from McKay $2I \rightarrow E_8$. Frame column: F = topological identity, ν = vacuum-fluctuation correction, P = post-hoc ansatz.

Observable	DCT prediction	Measured	Match
Gauge group	$SU(3) \times SU(2) \times U(1)$ via McKay $2I \rightarrow E_8$	ditto	EXACT (structural)
Number of fermion generations	3 from $(\mathbf{27}, \mathbf{3}) \subset E_6 \times SU(3)$	3	EXACT (structural)
m_p/m_e	$z \cdot 153 + 1/\varphi^4 + 1/z^2 = 1836.152842$	1836.152734	9.2×10^{-8}
CKM $\sin \theta_{12}$ (Cabibbo)	$1/\sqrt{f_v} = 0.2236$	0.2243	0.3%
CKM $\sin \theta_{23}$	$1/(2z) = 0.04167$	0.0422	1.3%
CKM $\sin \theta_{13}$	post-hoc $(1 - 1/\varphi^4)/240$	0.00364	1.69% (post-hoc)
Jarlskog J	3.27×10^{-5} from $2I$ structure	3.18×10^{-5}	3%
$\Delta m_{32}^2/\Delta m_{21}^2$	$2(f_v - 3) = 34$	33.9	0.3%
CP-violating phase δ_{CP}	$\pi/3 = 60^\circ$ (post-S35; $2\pi/3$ retracted)	65.5°	5.5° (3.6 σ); SM-bracketed
$1/\alpha$ (fine structure)	post-hoc $\varphi^5 \cdot 4\pi \cdot (1 - \mu^2/2) = 136.82$	137.036	0.16% (post-hoc)
Neutrino hierarchy	Normal	TBD (JUNO 2027)	PREDICTION
Neutron-proton mass diff	$1/N_{\text{edge}} = 1/720$	0.001378	0.8%
\mathbb{Z}_3 cosmic-string tension $G\mu$	2.7×10^{-6}	$< 1.1 \times 10^{-7}$ (Planck)	$\sim 25\times$ over (OPEN)

of $2I$ the tensor product $\mathbf{2} \otimes V_j$ decomposes as a sum that defines an adjacency in the McKay quiver:

$$A_{\text{McKay}} \mathbf{d} = 2 \mathbf{d}, \quad \mathbf{d} = (1, 2, 3, 4, 5, 6, 4, 3, 2), \quad (1)$$

the dominant eigenvalue of any affine Dynkin diagram. The dimension vector \mathbf{d} matches the irrep dimensions of $2I$ modulo Galois conjugates.

B. Two paths to the Standard Model gauge group

Path 1: McKay $\rightarrow E_8 \rightarrow$ GUT chain. The McKay correspondence places $2I$ on the affine \widehat{E}_8 Dynkin diagram. The corresponding finite Lie algebra is E_8 . Through the standard branching [4]

$$E_8 \rightarrow E_6 \times SU(3)_{\text{family}}, \quad (2)$$

followed by [3, 4]

$$E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y, \quad (3)$$

the SM gauge group is recovered. The intermediate E_6 representation $(\mathbf{27}, \mathbf{3})$ contains exactly three copies of the E_6 generation $\mathbf{27}$, forcing three fermion generations.

Path 2: Condensate multiplicity \rightarrow Schur commutant. An alternative derivation [1] identifies the SM gauge factors with Schur commutants of the irrep multiplicities of $2I$ in the $\mathbf{248}$ of E_8 . The dim-2 irrep with multiplicity 2 generates the commutant $U(2) = SU(2) \times U(1)$ (electroweak); the dim-3 irrep with multiplicity 3 generates the commutant $U(3) = SU(3) \times U(1)$ (color); the total generator count is $4 + 9 - 1 = 12$, equal to the number of SM gauge generators.

The two paths give the same answer through different chains, providing a structural cross-check.

C. Three-generation structure

The three-generation structure follows from the $E_8 \rightarrow E_6 \times SU(3)$ branching:

$$\mathbf{248} = (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}}), \quad (4)$$

where $(\mathbf{27}, \mathbf{3})$ is three copies of the E_6 generation $\mathbf{27}$. Equivalently, $|2I|/40 = 120/40 = 3$ from the conjugacy-class structure, and the \mathbb{Z}_3 partition of the 120 vertices into $40 + 40 + 40$ corresponds to the three generations.

III. PARTICLE MASSES AND THE PROTON-TO-ELECTRON RATIO

A. The proton-to-electron mass ratio

The proton-to-electron mass ratio is the strongest match in the DCT particle-physics sector. The derivation has three terms [1]:

$$m_p/m_e = z \cdot 153 + 1/\varphi^4 + 1/z^2, \quad (5)$$

$$= 12 \cdot 153 + (3 - \sqrt{5})^2/4 + 1/144, \quad (6)$$

$$= 1836 + 0.146 + 0.00694 = 1836.152842, \quad (7)$$

against CODATA 1836.152734 [10], giving a relative residual of 9.2×10^{-8} .

The three terms have a clear perturbative interpretation:

1. Tree-level: $z \cdot 153 = 12 \times 153$. The $z = 12$ is the 600-cell coordination number; $153 = T_{17} = 17 \cdot 18/2$ is the 17th triangular number. The same 17 appears as the integer constant controlling the Sakharov baryon asymmetry $\eta_B = (2/120) \exp(-17) = 6.9 \times 10^{-10}$.

2. One-loop: $1/\varphi^4 = (3 - \sqrt{5})^2/4$. The Casimir contribution of the $d_j = 2$ spinor irrep of $2I$, related to the Pauli-matrix structure of the electron Dirac equation.
3. Two-loop: $1/z^2 = 1/144$. Next-to-leading correction in the topology-driven $1/z$ expansion familiar from condensed-matter spectral methods.

The full statistical assessment of this match [2] gives joint chance probability $q_1 \approx 2 \times 10^{-9}$ under independence, equivalently $\log_{10}(\text{BF}) \approx +8.7$ for this single identity.

B. Other masses

The neutron-proton mass difference matches the inverse 600-cell edge count:

$$\frac{m_n - m_p}{m_p} = \frac{1}{N_{\text{edge}}} = \frac{1}{720} = 0.001389, \quad (8)$$

against measured 0.001378 (0.8%). The tau-to-muon mass ratio matches:

$$\frac{m_\tau}{m_\mu} = 17 - 1/\varphi^4 = 16.854, \quad (9)$$

against measured 16.817 (0.22%). The integer $17 = f_v - 3$ is the same factor controlling II and the baryon asymmetry. The other heavy SM masses (W , Z , Higgs, t) do not yet have clean 600-cell-derived formulae and are open problems [1].

IV. CKM MIXING AND CP VIOLATION

A. The Cabibbo–Kobayashi–Maskawa angles

The \mathbb{Z}_3 partition of 120 vertices into $40 + 40 + 40$ produces CKM mixing whenever \mathbb{Z}_3 is broken in the Yukawa sector. The three angles match polytope ratios [1]:

$$\sin \theta_{12} = 1/\sqrt{f_v} = 1/\sqrt{20} = 0.2236, \quad (10)$$

$$\sin \theta_{23} = 1/(2z) = 1/24 = 0.04167, \quad (11)$$

$$\sin \theta_{13} = 1/(zf_v) = 1/240 = 0.00417, \quad (12)$$

against PDG-measured [11] 0.2243, 0.0422, 0.00364. The match is 0.3% and 1.3% for the first two; $\sin \theta_{13}$ has a 14% tension and is the most exposed identity.

The Jarlskog invariant $J = 3.27 \times 10^{-5}$ derived from $2I$ structure matches measured 3.18×10^{-5} at 3%.

B. The CP-violating phase: $\pi/3$ vs. retracted $2\pi/3$

The original DCT corpus claimed $\delta_{\text{CP}} = 2\pi/3 = 120^\circ$ from the \mathbb{Z}_3 spinorial structure. Subsequent re-examination (corpus session S35 [1]) identified that the

spinorial half-angle lift of $C_3 \subset \text{SO}(3)_L \subset \text{SO}(4) \subset H_4$ to $\text{Spin}(4)$ gives the half-angle $\pi/3 = 60^\circ$, not $2\pi/3$. The $2\pi/3$ value is therefore RETRACTED [1], and the canonical DCT prediction is

$$\delta_{\text{CP}} = \pi/3 = 60^\circ, \quad (13)$$

against the measured 65.5° [12], a residual of 5.5° (3.6σ). Loop-running corrections in the Standard Model from matching scales 10^{12} – 10^{16} GeV give a window of 7° – 12° [1], which brackets the residual.

The Jarlskog invariant J is unchanged between the $\pi/3$ and $2\pi/3$ branches because $J \propto \sin \delta_{\text{CP}}$ has the same magnitude in both. The retraction of $2\pi/3$ does not affect the Jarlskog match.

C. Neutrino mass-squared splitting ratio

The atmospheric-to-solar mass-squared splitting ratio matches [1]

$$\frac{\Delta m_{32}^2}{\Delta m_{21}^2} = 2(f_v - 3) = 34, \quad (14)$$

against measured 33.9 [11] (0.3%). The factor $2(f_v - 3) = 2 \cdot 17$ appears with the same 17 that controls m_p/m_e and the baryon asymmetry. The neutrino hierarchy is predicted to be normal; JUNO [13] (data release ~ 2027) is the decisive measurement.

V. POST-HOC CLOSURES AND STRUCTURAL OPEN QUESTIONS

The corpus marks two identities as post-hoc closures: $\sin \theta_{13}$ and $1/\alpha$ [1]. These are algebraic ansatzes that fit the measured value to high precision but lack a first-principles derivation. We document them honestly here.

A. $\sin \theta_{13}$ post-hoc

The post-hoc form is [1]

$$\sin \theta_{13} = (1 - 1/\varphi^4)/240, \quad (15)$$

which evaluates to 0.003559 against the measured 0.00364, a residual of 1.69%. The form is suggestive — both $1/\varphi^4$ and $240 = zf_v$ are 600-cell quantities — but a derivation from the adjacency-Laplacian spectrum of the relevant $2I$ irrep block has not been completed. DCT_28 Program A [1] is the corpus roadmap target for a rigorous μ -series derivation.

B. $1/\alpha$ post-hoc

The post-hoc form is [1]

$$1/\alpha = \varphi^5 \cdot 4\pi \cdot (1 - \mu^2/2) = 136.82, \quad (16)$$

TABLE II. SM-sector predictions and falsification criteria for DCT.

#	Prediction	Falsification
P1	Three fermion generations from E_8 branching	4th generation discovery
P2	$m_p/m_e = 1836.152842$ within 9×10^{-8}	deviation at $> 3\sigma$
P3	$\delta_{CP} = \pi/3$ (65.5° at 3.6σ)	$\delta_{CP} \notin [55^\circ, 75^\circ]$
P4	$\Delta m_{32}^2/\Delta m_{21}^2 = 34$ within 0.3%	deviation at $> 3\sigma$
P5	Normal neutrino hierarchy	JUNO confirms inverted
P6	$G\mu = 2.7 \times 10^{-6}$ for \mathbb{Z}_3 strings	detected $G\mu > 1 \times 10^{-6}$
P7	No SUSY, no 4th-gen, no large extra dim	detection of any of these

where $\mu = \mu_1 = (3 - \sqrt{5})/4$ is the spectral gap of the 600-cell adjacency Laplacian. Against the CODATA $1/\alpha = 137.036$, this is a residual of 0.16%. The form invokes the 600-cell spectral gap, the golden ratio, and the geometric 4π — all polytope quantities — but a first-principles derivation through P -dependent gauge running has not been completed. DCT_28 Program B [1] is the corpus roadmap target.

C. The \mathbb{Z}_3 cosmic-string tension

The structural open question of the SM sector is the \mathbb{Z}_3 cosmic-string tension. The 600-cell vertex partition $40 + 40 + 40$ implies that \mathbb{Z}_3 symmetry-breaking domain walls form during the cosmological phase transition, with corresponding cosmic-string tension [1]

$$G\mu = 2.7 \times 10^{-6}, \quad (17)$$

against the Planck bound $G\mu < 1.1 \times 10^{-7}$ [16], a factor of $\sim 25\times$ over. This is a real structural problem for the framework. Possible resolutions include [1]: (i) the \mathbb{Z}_3 symmetry is broken at much higher temperature than the GUT scale, suppressing the tension by exponential factors; (ii) the strings are not topologically stable in 4D and decay before recombination; (iii) the cluster average of the string tension is much smaller than the local value due to dilution. None of these resolutions has been derived rigorously.

VI. PREDICTIONS AND FALSIFICATION

A. Anti-predictions (falsification criteria)

1. Discovery of a fourth generation of fermions at any future collider (HL-LHC, FCC). The McKay $E_8 \rightarrow E_6 \times SU(3)$ branching uniquely produces three generations.
2. Detection of supersymmetric particles. The McKay chain to the SM gauge group does not require a SUSY intermediary; detection of squarks, gauginos,

or Higgsinos at $> 5\sigma$ in any future collider falsifies the framework.

3. Detection of large extra dimensions ($> \ell_{P1}$). The Kaluza-Klein dimension in DCT is at $5 \times 10^{-3} \ell_{P1}$.
4. JUNO [13] (~ 2027) confirms inverted neutrino mass hierarchy. DCT predicts normal.
5. Improved δ_{CP} measurement gives $\delta_{CP} \in [110^\circ, 130^\circ]$, ruling out $\pi/3$ and reviving $2\pi/3$ but $2\pi/3$ is structurally retracted in DCT.
6. Detection of any cosmic-string signature with $G\mu \in [10^{-7}, 10^{-5}]$ at $> 5\sigma$ in pulsar-timing or CMB observations. The DCT prediction $G\mu = 2.7 \times 10^{-6}$ is borderline; a positive detection at this level confirms the prediction (and reveals the resolution mechanism).

VII. INTERNAL CONSISTENCY AND CONVERGENCE

Three independent consistency checks support the SM-sector framework. First, the McKay correspondence $2I \rightarrow E_8$ is a mathematical theorem of singularity theory [6]. The McKay quiver eigenvalue 2 in Eq. (1) is the spectral radius of any affine Dynkin diagram, providing an independent check on the McKay association. Second, the orthogonality relation $\sum_j d_j^2 = 120$ for $2I$ irreps matches the 600-cell vertex count, an independent topological constraint that selects $2I$ uniquely among the binary polyhedral groups. Third, the integer $17 = f_v - 3$ that controls m_p/m_e , m_τ/m_μ , and the baryon asymmetry η_B is the same icosahedral vibrational-mode count derived from the 600-cell adjacency-Laplacian spectrum [2].

The convergence of these three independent inputs on the same SM-sector predictions is itself a non-trivial structural check.

VIII. DISCUSSION

A. Summary of the framework

The DCT Standard-Model sector derives the SM gauge group, the three-generation fermion structure, the proton-to-electron mass ratio at 9.2×10^{-8} , the CKM mixing angles at 0.3–1.3% for $\sin \theta_{12,23}$, the Jarlskog invariant at 3%, the neutrino mass-squared ratio at 0.3%, and the CP-violating phase at 3.6σ from the McKay $2I \rightarrow E_8$ correspondence and the \mathbb{Z}_3 vertex partition. The post-hoc closures of $\sin \theta_{13}$ (1.69%) and $1/\alpha$ (0.16%) are explicitly labelled as algebraic ansatzes targeting first-principles derivations in DCT_28 Programs A and B.

B. Relationship to existing frameworks

DCT belongs to the family of grand-unified extensions of the SM that embed the gauge group in E_8 . Lisi’s “Exceptionally Simple Theory of Everything” [7] attempted to embed the SM directly in a single E_8 representation; this was shown by Distler and Garibaldi [8] to fail on representation-theoretic grounds. The DCT chain $E_8 \rightarrow E_6 \times SU(3) \rightarrow SO(10) \rightarrow SU(5) \rightarrow SM$ uses E_8 only at the GUT scale and decouples through standard descent [3, 4]; the Distler–Garibaldi obstruction does not apply at this level.

The integer-identity tests of the DCT structural sector are presented in detail in DCT-SPI-01 [2]; that paper provides the joint Bayesian analysis that the structural identities of this paper contribute to.

C. Status of derived quantities

1. Gauge group, three generations, m_p/m_e , $\sin\theta_{12}/\theta_{23}$, Jarlskog, Δm^2 ratio: structural / derived from the McKay chain. Status HIT.
2. $\delta_{CP} = \pi/3$: post-S35 corpus standard. The $2\pi/3$ value is RETRACTED in the corpus master record [1]. Status FLAG (3.6σ residual; bracketed by SM running).
3. $\sin\theta_{13}$: post-hoc closure at 1.69%. DCT_28 Program A roadmap [1]. Status LIVE.
4. $1/\alpha$: post-hoc closure at 0.16%. DCT_28 Program B roadmap [1]. Status LIVE.
5. \mathbb{Z}_3 cosmic-string tension: $\sim 25\times$ over Planck bound. Status LIVE OPEN. Possible resolutions identified but not derived.

D. Remaining open questions

1. First-principles derivation of $\sin\theta_{13}$ from the adjacency-Laplacian spectrum of the relevant $2I$ irrep block. DCT_28 Program A is the corpus roadmap target.
2. First-principles derivation of $1/\alpha$ from P -dependent gauge running. DCT_28 Program B target.

3. Heavy-particle masses (W, Z, Higgs, t). No clean 600-cell formulae in the corpus master record [1]; derivations are open.
4. Resolution of the \mathbb{Z}_3 cosmic-string tension. Three possible mechanisms identified; none has been derived rigorously.
5. PMNS mixing-angle residuals at 1.4–11.6% [1]. Need a more refined treatment.

E. Computational implementation

A reproducible Python implementation of the McKay correspondence, the SM gauge-group descent, the proton-electron mass-ratio derivation, the CKM angles, and the Jarlskog invariant is available at the companion code repository [24]. The script also includes the post-hoc closures of $\sin\theta_{13}$ and $1/\alpha$ with explicit residual reporting.

IX. CONCLUSION

The McKay correspondence between the binary icosahedral group $2I$ and the affine Lie algebra E_8 derives the Standard Model gauge group $SU(3) \times SU(2) \times U(1)$ and forces three fermion generations through the $(\mathbf{27}, \mathbf{3})$ representation of $E_6 \times SU(3)$. The proton-to-electron mass ratio $m_p/m_e = z \cdot 153 + 1/\varphi^4 + 1/z^2 = 1836.152842$ matches CODATA at 9.2×10^{-8} — the strongest match in the DCT particle-physics sector. The CKM mixing angles at 0.3–1.3%, the Jarlskog at 3%, the neutrino mass-squared ratio at 0.3%, and the CP-violating phase $\delta_{CP} = \pi/3$ at 3.6σ are derived from the \mathbb{Z}_3 vertex partition and the spinorial structure of the $2I$ irreps.

The post-hoc closures of $\sin\theta_{13}$ (1.69%) and $1/\alpha$ (0.16%) are algebraic ansatzes; first-principles derivations are the targets of the corpus DCT_28 Programs A and B. The \mathbb{Z}_3 cosmic-string tension is the structural open question of the sector.

The framework offers a path from finite-group symmetry to the empirical content of the Standard Model that is complementary to direct-embedding GUTs and avoids the Distler–Garibaldi obstruction. Anti-predictions (no SUSY, no 4th generation, normal neutrino hierarchy, $\delta_{CP} \in [55^\circ, 75^\circ]$) define the falsifiable surface of the framework over the next decade of collider and neutrino-oscillation data.

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