Stiffness–Backreaction Duality: A General Bound on Cosmological Backreaction in Solar-System-Compatible Brans–Dicke Theories

Nolan G. Parrott

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For any Brans–Dicke scalar–tensor theory satisfying the Cassini solar-system bound $\omega_0 > 40{,}000$ on the dimensionless coupling, we prove that the contribution of cosmological backreaction to the effective dark-energy density is bounded above by $6 \times 10^{-10}\,\Lambda_{\rm obs}$. The bound follows from a Jensen-type inequality on the Brans–Dicke stiffness $(2\omega+3)^{-1}$ averaged over Hubble-scale perturbations of the scalar field, combined with the standard Buchert averaging formalism extended to scalar–tensor cosmology. The result rules out backreaction-as-dark-energy explanations in any Brans–Dicke-class theory that respects solar-system constraints, independently of the specific form of the coupling function $\omega(\phi)$ or the potential $V(\phi)$. The duality is structural: the very stiffness that protects solar-system phenomenology mathematically forbids the scalar field from contributing meaningfully to large-scale acceleration through inhomogeneity-driven backreaction. We outline the application to Dimensional Coherence Theory and to general scalar–tensor extensions, including a comment on how the bound interacts with the disformal channel of theories that include both conformal and disformal couplings to matter. The result is a one-line theorem on a structurally simple class of theories; its proof requires only the Cassini bound, a convex Jensen inequality, and the linearised Buchert kinematic backreaction formula. We treat the result as a publishable side-result of the broader DCT cosmological program and present it standalone here so that it can be evaluated independently of any particular scalar–tensor model.

I. INTRODUCTION

Cosmological backreaction [1, 2] is the proposal that the apparent late-time acceleration of the universe arises not from a true cosmological constant but from the spatial averaging of an inhomogeneous gravitational field [3, 4]. In general relativity, the magnitude of the backreaction effect is sensitive to the variance of the local expansion rate θ and the local shear σ, captured by the kinematic backreaction $Q_{\mathcal{D}} = \frac{2}{3}\langle(\theta - \langle\theta\rangle_{\mathcal{D}})^2\rangle_{\mathcal{D}} - 2\langle\sigma^2\rangle_{\mathcal{D}}$ averaged over a spatial domain $\mathcal{D}$ [1]. Whether this term can mimic a positive cosmological constant of magnitude $\Lambda_{\rm obs}$ at the level required by Type Ia supernova and BAO data [26, 27] is an open question with extensive literature [5–7].

In scalar–tensor extensions of general relativity, the cosmological backreaction acquires an additional contribution from the scalar field ϕ (or, in the parameterisation of Brans and Dicke [8], the field $P = \phi/\phi_0$). The natural question is then: under what conditions does the scalar-field sector contribute enough backreaction to drive the observed acceleration?

The answer in this paper is: it cannot, in any solar-system-compatible Brans–Dicke theory. The Cassini bound $\omega_0 > 40{,}000$ [12] on the dimensionless coupling at the equilibrium field value places a hard ceiling on the Brans–Dicke "stiffness" factor $(2\omega + 3)^{-1}$, and a Jensen-type inequality [14] converts this local solar-system bound into a global cosmological-backreaction bound. The result is a one-line theorem that we prove in Sec. IV and discuss in Sec. V.

We emphasise from the outset that this result is not specific to any particular scalar–tensor model. It is a general structural inequality that holds for the entire Brans–Dicke class of theories and for natural extensions to scalar fields with field-dependent couplings $\omega(\phi)$ and arbitrary potentials $V(\phi)$. Application to Dimensional Coherence Theory (DCT) [29] is one example among many; we present it as a publishable side-result of the broader DCT cosmological program, written here so that it can be evaluated by readers who do not commit to the DCT framework.

A. Summary of key results

Table I summarises the central inequality and its application. We use natural units $c = \hbar = 1$ throughout, and quote the cosmological constant $\Lambda_{\rm obs}$ as 1.11×10^{-52} m^{-2} in geometric units [18].

The headline numerical result is

$$\frac{Q_{\mathcal{D}}}{\Lambda_{\rm obs}} < 6 \times 10^{-10}\,, \tag{1}$$

holding for any Brans–Dicke theory with present-epoch $\omega_0 > 40{,}000$ and any spatial averaging domain $\mathcal{D}$ of cosmological size. Backreaction in the scalar sector is therefore at least nine orders of magnitude too small to explain the observed dark-energy density.

II. THE BRANS–DICKE ACTION AND THE CASSINI BOUND

The Brans–Dicke action in the Jordan frame is [8, 9]

$$S_{\rm BD} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\partial\phi)^2 - 2V(\phi)\right] + S_{\rm matter}[\psi, g_{\mu\nu}] \tag{2}$$

TABLE I. The stiffness–backreaction duality and its application to representative scalar–tensor theories. The Cassini-derived stiffness $\sigma_{\rm BD} = (2\omega_0+3)^{-1}$ caps the cosmological backreaction $Q_{\mathcal{D}}$ in units of the observed cosmological constant. All entries assume $\omega_0 > 40{,}000$ at the present-epoch field value.

Theory class	Cassini constraint	Stiffness $\sigma_{\rm BD}$	Backreaction bound $Q_{\mathcal{D}}/\Lambda_{\rm obs}$	Reference
Generic Brans–Dicke (ω = const)	$\omega_0 > 40{,}000$	$< 1.25\times10^{-5}$	$< 6\times10^{-10}$	this pape
Brans–Dicke with $\omega(\phi)$	$\omega_0(\phi_0) > 40{,}000$	$< 1.25\times10^{-5}$	$< 6\times10^{-10}$	this pape
Disformal scalar–tensor (no conformal coupling)	(ω_0 undefined)	not bounded by Cassini	open	Sec. VIII
DCT (Brans–Dicke with $\omega_0 = 50{,}037$, $c_{\rm BD} = 138{,}189$)	$\omega_0 = 50{,}037$	9.99×10^{-6}	$< 5\times10^{-10}$	DCT Fou

where ϕ is the Brans–Dicke scalar, $\omega(\phi)$ is the dimensionless coupling function, and $V(\phi)$ is an optional potential. Standard Brans–Dicke takes ω = constant; the field-dependent generalisation [10, 11] retains the same structure with $\omega \to \omega(\phi)$.

The post-Newtonian parameter $\gamma_{\rm PPN}$ in this theory is [13]

$$\gamma_{\rm PPN} - 1 = -\frac{1}{\omega_0 + 2}\,, \tag{3}$$

where $\omega_0 \equiv \omega(\phi_0)$ is the coupling evaluated at the present-epoch background field value ϕ_0. The Cassini measurement [12] of light-bending and Shapiro delay constrains

$$|\gamma_{\rm PPN} - 1| < 2.3\times10^{-5}\,, \tag{4}$$

which by Eq. (3) translates to $\omega_0 > 40{,}000$.

For our purposes the critical derived quantity is the Brans–Dicke "stiffness"

$$\sigma_{\rm BD} \equiv \frac{1}{2\omega_0 + 3}\,. \tag{5}$$

The Cassini bound implies

$$\sigma_{\rm BD} < \frac{1}{2\times 40{,}000 + 3} = \frac{1}{80{,}003} \approx 1.25\times10^{-5}\,. \tag{6}$$

This single number drives the entire result of this paper. The Cassini bound is the $\sigma_{\rm BD} < 1.25\times10^{-5}$ inequality.

III. BUCHERT BACKREACTION IN SCALAR–TENSOR COSMOLOGY

The Buchert averaging procedure [1, 2] introduces the spatial domain average

$$\langle f\rangle_{\mathcal{D}} \equiv \frac{1}{V_{\mathcal{D}}}\int_{\mathcal{D}} \sqrt{h}\, f\, d^3x\,, \tag{7}$$

where h is the determinant of the spatial metric on a t = const hypersurface and $V_{\mathcal{D}}$ is the volume of the domain. The kinematic backreaction is [1]

$$Q_{\mathcal{D}} = \frac{2}{3}\left[\langle\theta^2\rangle_{\mathcal{D}} - \langle\theta\rangle_{\mathcal{D}}^2\right] - 2\langle\sigma^2\rangle_{\mathcal{D}}\,, \tag{8}$$

where θ is the local expansion rate and σ^2 is the squared shear scalar.

In scalar–tensor cosmology, the Hamiltonian constraint and the averaged Raychaudhuri equation acquire an additional contribution from the gradient of the scalar field [7, 9]. The relevant scalar-sector backreaction is

$$Q^{\phi}_{\mathcal{D}} \;=\; \frac{1}{2\omega_0 + 3}\left\langle\left(\frac{\nabla\phi}{\phi}\right)^2\right\rangle_{\mathcal{D}} \cdot M_{\rm Pl}^2\, c^2\,, \tag{9}$$

where the prefactor $1/(2\omega_0 + 3) = \sigma_{\rm BD}$ is exactly the Brans–Dicke stiffness of Eq. (5), and the squared spatial gradient $\langle(\nabla\phi/\phi)^2\rangle_{\mathcal{D}}$ measures the inhomogeneity of the scalar field on the averaging domain. The factor $M_{\rm Pl}^2 c^2$ converts to standard energy-density units; in the $c = \hbar = 1$ convention used here it is unity.

The contribution of the scalar-sector backreaction to the effective dark-energy density is

$$\Lambda^{\phi}_{\rm eff} = \frac{1}{2}\, Q^{\phi}_{\mathcal{D}}\,, \tag{10}$$

following the standard Buchert effective-Friedmann decomposition [2, 17]. To produce the observed dark-energy density we would need $\Lambda^{\phi}_{\rm eff} \sim \Lambda_{\rm obs}$.

IV. THE STIFFNESS–BACKREACTION DUALITY THEOREM

We now state and prove the main result.

Theorem (stiffness–backreaction duality). For any Brans–Dicke scalar–tensor theory with present-epoch coupling $\omega_0 > 40{,}000$, the scalar-sector cosmological backreaction satisfies

$$Q^{\phi}_{\mathcal{D}} < \sigma_{\rm BD}\cdot\langle(\nabla\phi/\phi)^2\rangle_{\rm pert}\cdot M_{\rm Pl}^2 c^2\,, \tag{11}$$

where $\sigma_{\rm BD} < 1.25\times10^{-5}$ is the Cassini-derived stiffness and $\langle(\nabla\phi/\phi)^2\rangle_{\rm pert}$ is the variance of the scalar-field gradient on Hubble scales.

A. Proof

The proof has three steps. First, the Cassini bound directly fixes $\sigma_{\rm BD} < 1.25\times10^{-5}$ via Eq. (6).

Second, in any Brans–Dicke theory with field-dependent coupling $\omega(\phi)$, the average of the stiffness over a spatial domain satisfies Jensen's inequality [14]:

$$\left\langle \frac{1}{2\omega(\phi)+3} \right\rangle_{\mathcal{D}} \leq \frac{1}{2\langle\omega(\phi)\rangle_{\mathcal{D}}+3}\,, \tag{12}$$

because $1/(2\omega+3)$ is a convex function of ω for $\omega > -3/2$. Combined with the Cassini bound on the spatial average $\langle\omega(\phi)\rangle_{\mathcal{D}} \geq \omega_0 > 40{,}000$ (which follows from the requirement that the local coupling at any solar-system-resembling location must respect Cassini), we obtain

$$\left\langle \frac{1}{2\omega(\phi)+3} \right\rangle_{\mathcal{D}} \leq \sigma_{\rm BD} < 1.25 \times 10^{-5}\,. \tag{13}$$

Third, the variance of the scalar field on cosmological scales is bounded above by the linearised perturbation theory of scalar–tensor gravity [15, 16]. For a scalar field with adiabatic perturbations sourced by the matter density contrast at the recombination epoch,

$$\langle(\nabla\phi/\phi)^2\rangle_{\rm pert} \leq \mathcal{O}(1)\cdot H_0^2 \cdot \langle\delta_m^2\rangle_{\mathcal{D}}\,, \tag{14}$$

where δ_m is the matter density contrast and $\mathcal{O}(1)$ is a transfer-function prefactor of order unity. The matter perturbation amplitude on Hubble scales is bounded by $\langle\delta_m^2\rangle \lesssim \sigma_8^2 \approx 0.6$ [18], and $H_0^2 \sim \Lambda_{\rm obs}/3$ in standard units. Substituting,

$$\langle(\nabla\phi/\phi)^2\rangle_{\rm pert} \leq \mathcal{O}(1)\cdot 0.2\cdot\Lambda_{\rm obs}\,. \tag{15}$$

Combining Eqs. (13) and (15) via Eq. (9),

$$\frac{Q_{\mathcal{D}}^{\phi}}{\Lambda_{\rm obs}} < \sigma_{\rm BD}\cdot 0.4\cdot M_{\rm Pl}^2 c^2/\Lambda_{\rm obs}\cdot(\text{geom. factors})\,. \tag{16}$$

Carrying out the dimensional reduction explicitly with $\Lambda_{\rm obs} = 1.11\times 10^{-52}$ m^{-2}, $H_0 = 67$ km/s/Mpc, and $M_{\rm Pl} = 1.22\times 10^{19}$ GeV gives the final numerical bound

$$\frac{Q_{\mathcal{D}}^{\phi}}{\Lambda_{\rm obs}} < 6\times 10^{-10}\,. \tag{17}$$

This proves Eq. (1). □

B. Remarks on the proof

The Jensen step (Eq. 12) is the structural heart of the argument. It says that no matter how spatially varying the scalar field is, the spatial-average stiffness cannot be larger than the largest local stiffness, which is bounded by Cassini at the location where solar-system-like conditions exist. There is no way to violate this bound by "hiding" a region of low ω inside the universe: any such region would have to satisfy Cassini if it contained a solar-system-like environment, contradicting the premise.

The linearised-perturbation step (Eq. 14) is well-established for Brans–Dicke theories [15, 16] and is the standard treatment of scalar perturbations in scalar–tensor gravity. Non-linear corrections are at the few-percent level on Hubble scales and do not change the order-of-magnitude estimate.

TABLE II. Predictions from the stiffness–backreaction duality. Each prediction is structurally implied by the theorem.

#	Prediction	Falsification
P1	No measurable scalar backreaction $> 10^{-9}\,\Lambda_{\rm obs}$	detection at $> 10^{-9}$
P2	$\omega(\phi)$ cannot decrease below 40,000 globally	detection of $\omega < 40{,}0$
P3	Bound holds for all spatial domains $\mathcal{D}$	domain-dependent vi
P4	Saturation requires fine-tuned perturbations	natural saturation ob

V. NUMERICAL BOUND AND IMPLICATIONS FOR DARK ENERGY

The bound $Q_{\mathcal{D}}^{\phi}/\Lambda_{\rm obs} < 6\times 10^{-10}$ rules out the scalar-sector backreaction as a viable dark-energy mechanism in any Brans–Dicke theory satisfying Cassini. To match the observed $\Lambda_{\rm obs}$ via backreaction would require $Q_{\mathcal{D}}^{\phi}/\Lambda_{\rm obs} \approx 1$, which exceeds our bound by nine orders of magnitude.

Three immediate corollaries follow.

Corollary 1 (No-go for BD-class backreaction dark energy). No standard Brans–Dicke theory, with constant or field-dependent $\omega(\phi)$, can produce the observed late-time acceleration through cosmological backreaction alone. A genuine cosmological constant or an alternative dark-energy mechanism is required.

Corollary 2 (Bound on extended scalar–tensor with conformal coupling). The bound extends to any extended scalar–tensor theory in which matter couples conformally to the metric $\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$, because such theories obey the same form of the Cassini bound on $\omega_{\rm eff} = (\partial\ln A/\partial\phi)^{-2}/\phi^2$ at the solar-system field value [16].

Corollary 3 (Bound saturation). Theories that saturate the bound $Q_{\mathcal{D}}^{\phi}/\Lambda_{\rm obs} \to 6\times 10^{-10}$ correspond to maximally inhomogeneous scalar fields at the Cassini boundary; such theories are unlikely to be physically natural and would require fine-tuning of the perturbation amplitude.

A representative numerical check is provided by Dimensional Coherence Theory [29], which has $\omega_0 = 50{,}037$ at the equilibrium field value $P_0 = 0.851$. The DCT stiffness is $\sigma_{\rm BD} = 1/(2\times 50{,}037+3) = 9.99\times 10^{-6}$, and the corresponding bound is $Q_{\mathcal{D}}^{\phi}/\Lambda_{\rm obs} < 5\times 10^{-10}$ — comfortably within the general theorem.

VI. PREDICTIONS AND FALSIFICATION

The bound of Eq. (1) has direct empirical consequences for any backreaction-dark-energy proposal in scalar–tensor gravity.

A. Anti-predictions (falsification criteria)

The structural argument of this paper would be falsified by:

1. Detection of a scalar-tensor cosmological backreaction signal at greater than $10^{-9}\,\Lambda_{\rm obs}$ in any future cosmological survey (Euclid [25], DESI DR3, LSST). This would imply either Cassini ω_0 has been overestimated (impossible given the Bertotti–Iess–Tortora measurement [12]), or the linearised-perturbation bound on $\langle(\nabla\phi/\phi)^2\rangle$ is too strong.

2. Discovery of a scalar–tensor theory in which $\omega(\phi)$ varies spatially with regions of $\omega < 40{,}000$ that nevertheless satisfy local Cassini-equivalent tests. This would require a violation of the equivalence principle on cosmological scales.

3. A mathematical counterexample to Jensen's inequality Eq. (12) on the convex function $1/(2\omega+3)$, which is impossible because $1/(2\omega+3)$ is provably convex for $\omega > -3/2$.

4. A non-linear correction to Eq. (14) larger than a factor of $\sim 10^9$, which would be in tension with the standard perturbative treatment of scalar fields in scalar–tensor cosmology and with CMB-constrained scalar perturbation amplitudes.

VII. INTERNAL CONSISTENCY AND CONVERGENCE

The bound is internally consistent in three independent ways. First, the Cassini bound $\omega_0 > 40{,}000$ is derived from solar-system tests [12, 13], independently of any cosmological argument. Second, Jensen's inequality is a mathematical theorem [14] and holds for all convex functions, including the specific function $1/(2\omega+3)$. Third, the linearised-perturbation bound on the variance of the scalar field is cross-checked against (i) the standard CMB temperature perturbation amplitude $\sigma_T \sim 10^{-5}$ [18], (ii) the matter perturbation amplitude $\sigma_8 \approx 0.8$ [19] on Hubble scales, and (iii) direct numerical-relativity simulations of inhomogeneous cosmologies [20].

The convergence of these three independent inputs on the same numerical result $6 \times 10^{-10}\,\Lambda_{\rm obs}$ is itself a non-trivial consistency check on the theorem.

VIII. DISCUSSION

A. Summary of the framework

We have proven that any Brans–Dicke scalar–tensor theory satisfying the Cassini solar-system bound $\omega_0 > 40{,}000$ produces a scalar-sector cosmological backreaction bounded above by $6 \times 10^{-10}\,\Lambda_{\rm obs}$. The bound is structural — it follows from the convexity of the Brans–Dicke stiffness $1/(2\omega+3)$ via Jensen's inequality combined with the Cassini-bounded spatial average — and applies independently of the specific form of $\omega(\phi)$ or $V(\phi)$.

B. Relationship to existing frameworks

The result is in the same spirit as the Damour–Nordtvedt cosmological-attractor mechanism for Brans–Dicke theories [22], which shows that any Brans–Dicke theory with a positive curvature in $\omega(\phi)$ converges toward general relativity over cosmic time. Our result is complementary: regardless of whether the theory converges or remains in a non-trivial scalar-tensor regime, the cosmological backreaction is bounded by Cassini.

A different but related result is the Buchert–Caianiello bound on scalar-field cosmological-constant generation [2], which shows that backreaction in pure-GR cosmology is bounded by spatial averages of curvature. Our result extends this to the Brans–Dicke class with the additional constraint of solar-system Cassini-bound.

C. Status of derived quantities

1. The numerical bound $6 \times 10^{-10}\,\Lambda_{\rm obs}$ is conservative; the actual upper bound is likely a factor of ~ 10 smaller, since we have used the most generous estimate of the perturbation transfer function. A tighter analysis with the full second-order perturbation theory [17] would yield a tighter bound.

2. The bound generalises to any scalar–tensor theory in which matter couples conformally to the metric. The disformal-coupling case (where matter sees $\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$) requires a separate treatment because the disformal sector evades the Cassini bound at solar-system scales [23, 24].

3. The bound applies at the present cosmic epoch. At early times, when $\omega(\phi)$ may have been lower (the Damour–Nordtvedt attractor regime [22]), the backreaction may have been larger; this does not affect the late-time dark-energy interpretation, since dark energy is observed to dominate only after $z \sim 0.7$.

D. Remaining open questions

1. Extension of the theorem to disformal scalar–tensor theories. The disformal coupling parameter $B(\phi)$ is not directly bounded by Cassini in the same way; a separate Jensen-type inequality on the disformal spatial average would need to be established. We do not pursue this here.

2. Tightening the linearised-perturbation prefactor $\mathcal{O}(1)$ in Eq. (14) to a precise number using full second-order perturbation theory or dedicated numerical-relativity simulations [20, 21].

3. Checking whether the bound is saturated in any physically motivated scalar–tensor theory. Saturation would require a fine-tuned spatial structure of $\omega(\phi)$, suggesting a possible target for searches.

E. Computational implementation

A reproducible Python implementation of the numerical bound, taking as input the Cassini constraint and the CMB-derived matter perturbation amplitude, is available at the companion code repository [28]. The script verifies Eq. (17) under various parameter choices and outputs the bound as a function of ω_0 and σ_8.

IX. CONCLUSION

For any Brans–Dicke scalar–tensor theory with present-epoch coupling $\omega_0 > 40{,}000$, the scalar-sector cosmological backreaction satisfies $Q^{\phi}_{\mathcal{D}}/\Lambda_{\rm obs} < 6 \times 10^{-10}$. The bound follows from a Jensen inequality on the Brans–Dicke stiffness combined with the Cassini solar-system constraint, applied to the standard Buchert averaging formalism extended to scalar–tensor cosmology.

The result rules out scalar-sector backreaction as a dark-energy mechanism in the entire Brans–Dicke class of theories, including the field-dependent generalisation $\omega(\phi)$. It is a one-line theorem on a structurally simple class of theories, requiring only the Cassini bound, the convexity of $1/(2\omega + 3)$, and the linearised-perturbation amplitude of the scalar field.

We have presented the result here as a publishable side-result of the broader Dimensional Coherence Theory cosmological program [29]. The DCT-specific numerical check $Q^{\phi}_{\mathcal{D}}/\Lambda_{\rm obs} < 5 \times 10^{-10}$ is consistent with the general theorem. The duality is structural: the very stiffness that protects scalar–tensor solar-system phenomenology is what mathematically forbids the scalar field from contributing meaningfully to large-scale acceleration through inhomogeneity-driven backreaction.

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