


# BAO background, null propagation, and the perturbation-level programme in Dimensional Coherence Theory

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Dimensional Coherence Theory (DCT) is a Brans–Dicke scalar–tensor framework with matter minimally coupled to the conformal physical metric  $\tilde{g}_{\mu\nu} = P g_{\mu\nu}$  ([2]). This paper **retracts** the historical background-level BAO argument based on multiplying comoving angular-diameter distances by  $1/\sqrt{P_0}$ : for spatially homogeneous  $P(t)$ , radial *null* photon worldlines in FLRW obey  $0 = P(-dt^2 + a^2 d\chi^2)$ , so  $d\chi/dt = 1/a$  and  $P$  drops out — the standard comoving ruler  $\chi(z)$  is unchanged. The published  $\Delta\chi^2 \approx 33.6$  “ $5.8\sigma$ ” figure therefore targeted a **geometrically inconsistent** distance map, not a prediction of the homogeneous conformal ansatz. Late-time operational statements such as  $H_{\text{phys}} = H_E/\sqrt{P_0}$  must be derived from matter proper time on  $\tilde{g}$  and must not be inserted into the same null-integral pipeline as an ad hoc inverse power of  $\sqrt{P}$ . Background BAO from homogeneous  $P$  alone is **degenerate** with  $\Lambda$ CDM at the level of  $\chi(z)$ ; tests migrate to inhomogeneous  $P$ , disformal channels, and the perturbation-level programme  $(\mu_b, \mu_{\text{DM}}, \Sigma)$  on a  $\Lambda$ CDM background. The Jensen inequality, applied to convex functionals of  $P$ , constrains *biased* or LOS-averaged observables built from  $1/\sqrt{P(\mathbf{x})}$ , not the homogeneous null comoving distance. The smoking-gun cluster-scale discriminator remains  $M_{\text{lens}}/M_{\text{dyn}}(z \sim 1.5) = 1.30$  ([1, 2]).

## I. INTRODUCTION

Dimensional Coherence Theory (DCT) couples matter to  $\tilde{g}_{\mu\nu} = P g_{\mu\nu}$  with Gross–Pitaevskii / 600-cell structure [2]. The historical claim that *homogeneous* background DCT rescales BAO comoving distances by  $1/\sqrt{P_0}$  while leaving  $r_d$  fixed is **geometrically inconsistent**: for spatially homogeneous  $P(t)$ , radial null rays satisfy  $d\chi/dt = 1/a(t)$  because  $P$  cancels from  $0 = P(-dt^2 + a^2 d\chi^2)$ . Thus standard photon comoving distances  $\chi(z)$  — and homogeneous-background BAO fits tied to them — are **not** shifted by an overall time-dependent conformal factor acting uniformly on  $-dt^2$  and  $a^2 d\chi^2$ . The legacy  $\Delta\chi^2 \approx 33.6$  audit figure implemented a mistaken distance map, not a theorem about null propagation in DCT.

Operational late-universe statements such as  $H_{\text{phys}} = H_E/\sqrt{P_0}$  (SH0ES-scale prose in [2]) must be grounded in matter proper time on  $\tilde{g}$ , not reverse-engineered into the null comoving integral with an ad hoc inverse power of  $\sqrt{P}$ .

This paper is structured to: (a) document the Friedmann sector and the null-cone cancellation, (b) restate what remains testable at background BAO (inhomogeneous  $P$ , disformal extensions, clock/redshift definitions), (c) summarise the Jensen bound as it applies to *positivity-weighted nonlinear functionals* of  $P(\mathbf{x})$  rather than to  $\chi_{\text{null}}$ , and (d) carry the perturbation-level programme and the cluster-scale  $M_{\text{lens}}/M_{\text{dyn}}$  discriminator forward unchanged.

### A. Summary of key results

The two most important quantitative results are:

$$\Delta\chi_{\text{BAO, legacy script}}^2 \approx +33.6 \quad (\text{obsolete distance map — not a physical prediction}) \quad (1)$$

the legacy audit  $\Delta\chi^2$  artefact computed from the mistaken  $D_M$  rescaling against DESI Y1 redshift bins [5] (not a null-cone prediction; see Sec. III), and

$$\frac{M_{\text{lens}}}{M_{\text{dyn}}}(z \sim 1.5) = 1.30, \quad (2)$$

the smoking-gun perturbation-level prediction for the lens-to-dynamical mass ratio, falsifiable by Euclid 2027–2029 [7].

## II. BACKGROUND-FRAME DCT AND THE CONFORMAL-FRAME MAPPING

In the canonical DCT action [2]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left[ PR - \frac{\omega(P)}{P} (\partial P)^2 - 2V(P) \right], \quad (3)$$

the Brans–Dicke amplitude  $P(x, t)$  has equilibrium value  $P_0 = 0.851$  at the present epoch and a coupling function  $\omega(P) = (cP^2 - 3)/2$  with  $c = 138,189$ , giving  $\omega_0 \equiv \omega(P_0) \approx 50,037$ . The conformal physical metric is  $\tilde{g}_{\mu\nu} = P g_{\mu\nu}$ , and the modified Friedmann equation in the Jordan frame yields

$$\left( H + \frac{\dot{P}}{2P} \right)^2 = \frac{8\pi G}{3} \rho. \quad (4)$$

Three screening mechanisms enforce  $P(x, t) = P_0$  at all redshifts  $z < 10^6$  [2]: the Brans–Dicke stiffness  $1/(2\omega_0 + 3) \sim 10^{-5}$ , the Avrami exponential screening, and the Yukawa cutoff at the field mass  $m \sim 65$  Mpc. As a

TABLE I. Status of background BAO under homogeneous conformal  $P(t)$  (this revision). The legacy audit script implemented a mistaken distance rescaling; null photon comoving distances are unchanged (Sec. III). The perturbation-level program operates on a  $\Lambda$ CDM background with linearised  $\mu_b(a)$ ,  $\mu_{\text{DM}}(a)$ , and  $\Sigma(a)$  kernels.

Theory	$H_0$ (km/s/Mpc)	BAO background vs. $\Lambda$ CDM	Tension	Sc
$\Lambda$ CDM	67.4 (Planck)	0 (reference)	—	7.5/10 (with)
Homogeneous $P(t)$ only, null $\chi$	67.4 in standard $\chi(z)$	<b>degenerate</b> at $\chi(z)$ level	none from $P$ alone	
Legacy mistaken map ( $D \rightarrow D/\sqrt{P_0}$ )	(not applicable)	produced audit $\Delta\chi^2$ artifact	<b>retracted</b>	obs
DCT perturbation-level	$\Lambda$ CDM background; $\mu_b(a) = 1/P$ etc.	$\sim +1$ (negligible)	not violated	5.5/10

consequence,  $\dot{P} = 0$  at all observable epochs and the Friedmann equation reduces to

$$H^2 = \frac{8\pi G}{3}\rho, \quad (5)$$

identical to general relativity. The Hubble rate measured locally in the physical frame is then [2]

$$H_{\text{phys}} = H_E/\sqrt{P_0}, \quad (6)$$

where  $H_E$  is the Einstein-frame value inferred from the CMB. With Planck [4]  $H_E = 67.4$  km/s/Mpc and  $\sqrt{P_0} = 0.9225$ , this gives  $H_{\text{phys}} = 73.06$  km/s/Mpc, matching SH0ES [3] at 0.04%.

### III. BAO BACKGROUND: NULL PROPAGATION AND THE RETIRED DISTANCE RESCALING

#### A. The DESI Y1 measurement

DESI Year-1 reports the BAO peak position  $D_M(z)/r_d$  and the Hubble distance  $D_H(z)/r_d$  at multiple redshifts in the range  $z = 0.295$  to  $z = 2.330$  [5]. The distance ratios are sensitive to both the comoving angular-diameter distance to the drag epoch and the sound-horizon scale at the drag epoch.

In  $\Lambda$ CDM with Planck cosmology, the sound horizon is  $r_d^{\Lambda\text{CDM}} = 147.09$  Mpc and the angular-diameter distances scale as  $D_M(z) = c/H_0 \cdot \chi(z)$  where  $\chi(z)$  is the standard comoving-distance integral. DESI Y1 reports  $\Delta\chi^2 = 0$  for  $\Lambda$ CDM Planck across all redshift bins (consistent within statistical uncertainty).

#### B. Homogeneous background DCT and comoving null distances

In the homogeneous, spatially flat FLRW sector write the line element as

$$ds^2 = P(t)(-dt^2 + a^2(t) d\chi^2 + a^2(t) d\Omega^2). \quad (7)$$

Radial photons obey  $0 = P(-dt^2 + a^2 d\chi^2)$ . Because  $P > 0$  everywhere,  $P$  divides out and

$$\frac{d\chi}{dt} = \frac{1}{a(t)}, \quad (8)$$

**identical** to the inner metric  $-dt^2 + a^2 d\chi^2$ . Integrating,

$$\chi(z) = \int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{c dt}{a(t)} = \int_0^z \frac{c dz'}{H(z')}, \quad (9)$$

with the **same**  $H(z)$  as in the Einstein-frame FLRW defined by  $g_{\mu\nu}$ , given Friedmann (5). Therefore **no** uniform multiplicative rescaling  $D_M \rightarrow D_M/\sqrt{P_0}$  or  $D_M \rightarrow \sqrt{P_0} D_M$  arises from homogeneous  $P(t)$  alone at the level of standard null-relative comoving distances. The legacy “ $D_M^{\text{DCT}} = D_M^{\Lambda\text{CDM}}/\sqrt{P_0}$ ” formula conflated operational  $H$ -statements with the null integral; it is retracted.

#### C. Legacy audit figure (not a theory test)

Repository script `dct.desi.bao.test.py` reproduced the large  $\Delta\chi^2$  by applying an empirical  $\sim 3.25\%$  multiplicative pull on  $D_M/r_d$  designed to mimic the mistaken  $1/\sqrt{P_0}$  rescaling against synthetic data. Because (8) forbids that rescaling for homogeneous  $P$ , the figure  $\Delta\chi^2 \approx 33.6$  is **not** interpretable as “DCT falsified at  $5.8\sigma$  by DESI” in the homogeneous conformal sector.

### IV. JENSEN BOUNDS ON BIASED $P(\mathbf{x})$ FUNCTIONALS (CONDITIONAL)

When  $P$  varies **spatially**, photon geodesics must be computed in the full metric  $ds^2 = P(t, \mathbf{x})(-dt^2 + a^2(t) \gamma_{ij} dx^i dx^j)$ ; there is no universal factorisation  $D_M^{\Lambda\text{CDM}} \cdot \langle 1/\sqrt{P} \rangle_{\mathcal{D}}$ . The legacy “spatially averaged” construction [14]

$$\langle D_M \rangle_{\mathcal{D}} \stackrel{\text{old}}{=} D_M^{\Lambda\text{CDM}} \cdot \left\langle \frac{1}{\sqrt{P}} \right\rangle_{\mathcal{D}} \quad (10)$$

was an **ansatz** for tracer-weighted observables, not a theorem about  $\chi_{\text{null}}$  from (8).

**Conditional Jensen.** Whenever a positive-weighted average of a *strictly convex* function  $f(P)$  is physically appropriate — e.g. certain density-weighted estimators — Jensen’s inequality [13] still yields  $\langle f(P) \rangle \geq f(\langle P \rangle)$ . This does **not**, after the retraction of (10) as a photon distance, imply an automatic  $+8.4\%$  shift on DESI  $D_M/r_d$  for arbitrary  $P(\mathbf{x})$ .

## A. Implications

Background tests of homogeneous  $P(t)$  against BAO must either (i) work at the level of **clock/redshift** and matter observables tied to  $\tilde{g} = Pg$ , or (ii) involve explicit ray tracing / Green's functions with inhomogeneous  $P$ . The perturbation-level programme [2] remains on a  $\Lambda$ CDM background with modified  $\mu_b$ ,  $\mu_{\text{DM}}$ , and  $\Sigma$ .

## V. THE PERTURBATION-LEVEL PROGRAM

In the perturbation-level program [2], the background cosmology is identical to  $\Lambda$ CDM (so the BAO success is automatic), and the DCT modifications appear only in the linearised growth and lensing kernels. Specifically, the modified Poisson equations are

$$k^2\Phi = -4\pi Ga^2\mu_b(a)\rho_b\delta_b - 4\pi Ga^2\mu_{\text{DM}}(a)\rho_{\text{DM}}\delta_{\text{DM}}, \quad (11)$$

$$k^2\Phi_{\text{WL}} = -4\pi Ga^2\Sigma(a)\rho_{\text{tot}}\delta_{\text{tot}}, \quad (12)$$

where  $\Phi$  is the Newtonian dynamical potential,  $\Phi_{\text{WL}}$  is the Weyl lensing potential, and the modification kernels are

$$\mu_b(a) = \frac{1}{P(a)}, \quad \mu_{\text{DM}}(a) = \frac{1}{P(a)(1+\beta)}, \quad \Sigma(a) = \frac{1}{\bar{P}(a)}, \quad (13)$$

with  $\beta = f_v/z = 5/3$  from the 600-cell vertex figure (canonical DCT) [2] and  $\bar{P}(a)$  the line-of-sight average of  $P(a)$  on the lensing path. At the present epoch,  $P(a) = P_0 = 0.851$ , so  $\mu_b = 1.175$ ,  $\mu_{\text{DM}} = 0.441$ , and  $\Sigma = 1.175$ .

The two-parameter dependence  $\mu_b - \mu_{\text{DM}}$  produces a non-trivial signature in galaxy-galaxy lensing and in the lens-to-dynamical mass ratio of clusters. The perturbation-level program score is 5.5/10 [2] on the corpus master scorecard, with the smoking-gun observable being the  $M_{\text{lens}}/M_{\text{dyn}}$  turnover.

### A. The $M_{\text{lens}}/M_{\text{dyn}}$ decisive test

The lens-to-dynamical mass ratio in a cluster of galaxies is the ratio of the lensing mass (sensitive to  $\Sigma$ ) to the dynamical mass (sensitive to  $\mu_b + \mu_{\text{DM}}$ ). In  $\Lambda$ CDM this ratio is unity at all redshifts. In the perturbation-level DCT program [2], the ratio peaks at 1.30 at  $z \sim 1.5$  as  $\Sigma$  exceeds the dynamical kernels:

$$\frac{M_{\text{lens}}}{M_{\text{dyn}}}(z) = \frac{\Sigma(z)}{(\mu_b(z) + \mu_{\text{DM}}(z))/2}, \quad (14)$$

which evaluates to 1.30 at  $z = 1.5$  and declines toward unity at  $z = 0$  and  $z > 2$ . Euclid is the most direct probe [7]; LSST and Roman cluster lensing programmes provide cross-checks.

TABLE II. Predictions and falsification criteria from the BAO no-go theorem and the perturbation-level program.

#	Prediction	Falsifi
P1	Legacy script $\Delta\chi_{\text{BAO}}^2 \approx 33.6$ (obsolete map)	supers
P2	Conditional Jensen on convex $f(P)$ for biased estimators only	misap
P3	$M_{\text{lens}}/M_{\text{dyn}}(z \sim 1.5) = 1.30$	no tur
P4	Pert. DCT $\Delta\chi_{\text{BAO}}^2 \sim +1$	pert. 1
P5	$\sigma_8 = 0.756$ , $S_8 = 0.775$	strong
P6	Splashback $R_{\text{sp}}/R_{200} = 0.923$	DES Y

## VI. PREDICTIONS AND FALSIFICATION

### A. Anti-predictions (falsification criteria)

The structural argument of this paper would be falsified by:

1. A fully covariant derivation linking  $H_{\text{phys}} = H_E/\sqrt{P_0}$  to meter sticks on  $\tilde{g} = Pg$  that is **compatible** with (8) for photon BAO.
2. Ray-traced background BAO with inhomogeneous  $P(\mathbf{x}, t)$  or disformal sector included — not the retracted linear rescaling.
3. A counterexample to standard photon null propagation in FLRW under homogeneous overall  $P(t)$ . None exists within metric (7).
4. Observation of a  $M_{\text{lens}}/M_{\text{dyn}}$  ratio that is uniformly unity across  $z = 0-2$ , ruling out the perturbation-level program.

## VII. INTERNAL CONSISTENCY AND CONVERGENCE

Three inputs now separate cleanly: (i) the canonical action (3) with matter on  $\tilde{g} = Pg$  [2]; (ii) null radial comoving distances (8) unchanged by homogeneous  $P(t)$ ; (iii) optional operational  $H_{\text{phys}} = H_E/\sqrt{P_0}$  prose, which must be derived from matter clocks and must not be reverse-engineered into the null  $\chi$  integral. The legacy  $\Delta\chi^2 \approx 33.6$  figure remains reproducible from the mistaken rescaling script [14] for audit archaeology only.

## VIII. DISCUSSION

### A. Summary of the framework

Homogeneous conformal rescaling of FLRW by  $P(t)$  does not shift standard photon comoving BAO rulers. The historical claim of a  $5.8\sigma$  DESI falsification from a uniform  $1/\sqrt{P_0}$  distance map is retracted as a geometric error. Live tests are: (a) explicit inhomogeneous

$P(\mathbf{x}, t)$  or disformal extensions at background level; (b) the perturbation-level programme [2] on a  $\Lambda$ CDM background; (c) cluster-scale  $M_{\text{lens}}/M_{\text{dyn}}$  turnover at  $z \sim 1.5$  peaking at 1.30.

### B. Relationship to existing frameworks

The Hubble-tension literature [8, 9] spans early- and late-universe solutions. DCT’s operational  $H_{\text{phys}}$  statements require a careful clock/redshift derivation on  $\tilde{g} = Pg$ ; they cannot be coupled to BAO through the retracted rescaling. The perturbation-level program is a member of the EFT of dark energy class [12], with  $\mu_b$ ,  $\mu_{\text{DM}}$ , and  $\Sigma$  as structure functions.

### C. Status of derived quantities

1. Homogeneous background BAO from  $P(t)$  alone: **degenerate** with  $\Lambda$ CDM at  $\chi(z)$  level (8); no universal DESI tension from the old map.
2. Perturbation-level DCT score: 5.5/10 [2], on a  $\Lambda$ CDM background with two parameters above  $\Lambda$ CDM.
3. Hubble-tension framing: requires revised derivation of how SH0ES connects to  $\tilde{g}$ ; not tied to retracted BAO rescaling.
4. Future Euclid DR1 [7] and DESI DR2 [5] tighten cluster-lensing and growth tests.

### D. Remaining open questions

1. Disformal-sector contribution to the BAO at perturbation level. The  $(1 - P)^2$  Avrami screening

factor in the disformal coupling [2] confines disformal effects to large scales and may shift the  $\sigma_8/S_8$  predictions in directions not yet computed.

2. The KiDS-Legacy 2025 [6] measurement  $S_8 = 0.815$  places DCT’s  $S_8 = 0.775$  at  $\sim 2\sigma$  tension; the perturbation-level program score of 5.5/10 already accounts for this, but a tighter measurement (Euclid DR1) could reduce it further.
3. The cosmic-chronometer per- $z$  ratio test contradicts simple BEC at  $3.08\sigma$  [2] (favours  $\Lambda$ CDM at  $0.63\sigma$ ); this is a real negative the perturbation-level program must address.

### E. Computational implementation

Scripts in the NGP.COM repository reproduce the **legacy** mistaken rescaling (`dct_desi_bao_test.py`) and document null-cone cancellation (`bao_conformal_null_check.py`, `dct_bd_frame_dictionary.py`). Companion [14] should be updated to match this revision.

## IX. CONCLUSION

Homogeneous  $P(t)$  in  $ds^2 = P(t)(-dt^2 + a^2 d\chi^2)$  leaves radial null comoving distances unchanged (8). The DESI background-BAO “5.8 $\sigma$ ” figure built from  $D_M \propto D_M^{\Lambda\text{CDM}}/\sqrt{P_0}$  targeted an inconsistent map, not a prediction of that geometry. Conditional Jensen inequalities apply only where a convex functional of  $P$  is **physically** appropriate — not to the retracted linear rescaling of photon  $\chi$ . The perturbation-level programme on a  $\Lambda$ CDM background, and cluster-scale  $M_{\text{lens}}/M_{\text{dyn}}$ , remain the falsifiable interfaces with data [1, 2].

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