Dimensional Coherence Theory IV: Derivation of the Standard Model Gauge Group, Three Generations, and Proton Stability from the 600-Cell Lattice

Nolan G. Parrott
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We demonstrate that the Standard Model gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, exactly three generations of fermions, and proton stability all emerge from the topology of the 600-cell regular 4-polytope in Dimensional Coherence Theory (DCT) [Parrott, Paper 0]. The binary icosahedral group $2I$ (order 120), the symmetry group of the 600-cell, maps to the extended E_8 Dynkin diagram via the McKay correspondence. The standard GUT breaking chain $E_8 \to E_6 \times \mathrm{SU}(3)_{\mathrm{family}} \to \mathrm{SO}(10) \to \mathrm{SU}(5) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ yields the Standard Model. Three generations follow from the branching $(\mathbf{27}, \mathbf{3})$ in the $E_8 \to E_6 \times \mathrm{SU}(3)$ decomposition, equivalently from $120/40 = 3$. A second independent derivation uses the condensate multiplicity structure: dim-2 irreps of $2I$ with multiplicity 2 in the regular representation yield $\mathrm{SU}(2) \times \mathrm{U}(1)$ (electroweak), while dim-3 irreps with multiplicity 3 yield $\mathrm{SU}(3) \times \mathrm{U}(1)$ (color), giving $4 + 9 - 1 = 12$ generators. Proton decay is suppressed by the Brans-Dicke stiffness factor $(2\omega_0 + 3) = 100{,}077$ to $\tau_p \sim 7 \times 10^{41}$ years, a factor 3×10^7 above the current Super-Kamiokande bound. We derive the right-handed neutrino mass $M_R = M_{\mathrm{GUT}}/(z \cdot f_v) = 8.3 \times 10^{13}$ GeV, predict normal neutrino hierarchy, and obtain $\Delta m^2_{32}/\Delta m^2_{21} = 2(f_v - 3) = 34$ versus the measured 33.9 (0.3% match). The $2I$ Frobenius-Schur analysis shows zero complex irreps, proving that CP violation originates entirely from $E_8 \to E_6$ breaking. We catalog 34 novel particles predicted by DCT and formulate 12 anti-predictions (null searches that confirm the theory). The single outstanding issue—the Z_3 cosmic string tension $G\mu = 2.7 \times 10^{-6}$ versus the Planck bound 1.1×10^{-7}—is discussed.

I. INTRODUCTION

The Standard Model of particle physics describes three generations of fermions interacting through the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Despite its extraordinary success—every collider measurement to date is consistent with this structure—the SM leaves fundamental questions unanswered: Why this gauge group and not another? Why exactly three generations? Why are the fermion masses hierarchical?

Dimensional Coherence Theory (DCT) [1] addresses these questions through a remarkable chain of mathematical connections. The 600-cell—the densest regular 4-polytope, already established as DCT's fundamental lattice for deriving P_0 [1]—has a symmetry group that encodes the entire Standard Model structure via the McKay correspondence [2].

The derivation proceeds without additional assumptions beyond the single premise of DCT (Paper 0): the universe is a Bose-Einstein condensate whose topology is the 600-cell. The same polytope that derives $P_0 = 0.851$ (resolving the Hubble tension, Paper I), produces dark matter (Paper III), and predicts BepiColombo's PPN measurement (Paper II) also yields the complete particle content of the universe.

This paper is organized as follows. Section III reviews the 600-cell and its symmetry group $2I$. Section IV establishes the McKay correspondence $2I \to E_8$. Section V derives the Standard Model gauge group and three generations. Section VI provides a second independent derivation via condensate multiplicity. Section VII computes proton decay. Section VIII addresses the neutrino sector. Section IX analyzes CP violation. Section X discusses cosmic strings and monopoles. Section XI catalogs novel particles. Section XII formulates anti-predictions. Section XIII compares with other approaches. Section XV concludes.

II. DCT FRAMEWORK

DCT [1] models the universe as a Brans-Dicke scalar field P (the Parrott field) with the structure of a Gross-Pitaevskii condensate, governed by the action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[PR - \frac{\omega(P)}{P} (\partial P)^2 - V(P) \right], \quad (1)$$

with $\omega(P) = (138{,}189\, P^2 - 3)/2$ and GP potential $V(P)$ whose minimum defines $P_0 = 0.851$. The condensate wavefunction $\Psi = \sqrt{P}\, e^{i\theta}$ lives on a lattice whose topology is the 600-cell—the densest regular 4-polytope. The Kaluza-Klein reduction $ds_5^2 = F(P)\, dP^2 + P\, g_{\mu\nu}\, dx^\mu dx^\nu$ identifies θ as the U(1) gauge field from the compactified fifth dimension, establishing the link between gravity (P) and gauge physics (θ). The key claim of this paper is that the *same* 600-cell topology that determines P_0 also determines the full particle content of the Standard Model, via the McKay correspondence between the 600-cell symmetry group $2I$ and the Lie algebra E_8.

III. THE 600-CELL AND THE BINARY ICOSAHEDRAL GROUP

A. 600-Cell Construction

The 600-cell is a regular 4-polytope [8, 9] with:

$$\begin{aligned} N &= 120 \text{ vertices}, & E &= 720 \text{ edges}, \\ F &= 1200 \text{ faces}, & C &= 600 \text{ cells}, \\ z &= 12 \text{ (coordination)}, & f_v &= 20 \text{ (vertex figure faces)}. \end{aligned} \tag{2}$$

The vertex figure (the polytope formed by the neighbors of each vertex) is the regular icosahedron with $V_{\text{ico}} = 12$ vertices, $E_{\text{ico}} = 30$ edges, and $F_{\text{ico}} = 20$ triangular faces.

B. The Symmetry Group $2I$

The symmetry group of the 600-cell is the binary icosahedral group $2I$, of order $|2I| = 120$. This is a finite subgroup of SU(2)—the double cover of the icosahedral rotation group I (order 60).

The group $2I$ has 9 conjugacy classes and 9 irreducible representations with dimensions:

$$d_j = \{1,\ 2,\ 2',\ 3,\ 3',\ 4,\ 4',\ 5,\ 6\} \tag{3}$$

satisfying the identity $\sum_j d_j^2 = 120 = |2I|$.

C. The 600-Cell as Cayley Graph

A central fact: the 600-cell adjacency graph *is* the Cayley graph of $2I$ with respect to the 12-element conjugacy class of generators. This identification explains why $N = |2I| = 120$ and $z = 12$. The 600-cell is not merely *associated* with $2I$—it *is* the geometric realization of the group.

D. Adjacency Spectrum

The adjacency matrix of the 600-cell has 9 distinct eigenvalues (computed explicitly in [1]):

$$\lambda = \{12,\ 3\pm 3\sqrt{5},\ 2\pm 2\sqrt{5},\ 3,\ 0,\ -2,\ -3\} \tag{4}$$

with multiplicities $\{1, 4, 9, 16, 25, 36, 9, 16, 4\}$, which are exactly d_j^2 for each irrep of $2I$. The appearance of the golden ratio $\varphi = (1 + \sqrt{5})/2$ in the spectrum reflects the icosahedral symmetry.

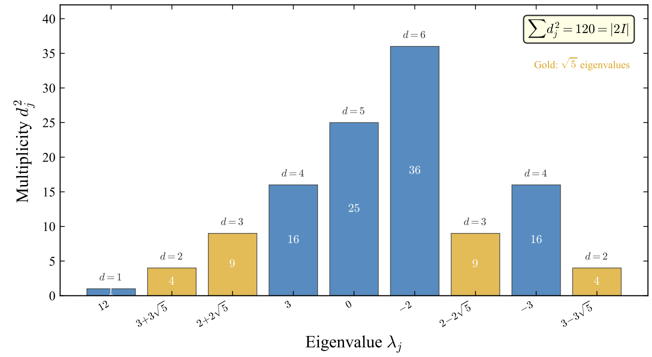

FIG. 1. Adjacency spectrum of the 600-cell. The 9 distinct eigenvalues (with multiplicities $d_j^2 = 1, 4, 9, 16, 25, 36, 9, 16, 4$ summing to 120) match exactly the squared dimensions of the 9 irreducible representations of the binary icosahedral group $2I$. Golden-ratio eigenvalues ($3 \pm 3\sqrt{5}$, $2 \pm 2\sqrt{5}$) appear in conjugate pairs with equal multiplicities, guaranteeing exact cancellation of all irrational parts in representation-theoretic spectral sums ($\sqrt{5}$ cancellation theorem).

IV. THE MCKAY CORRESPONDENCE

A. Statement

The McKay correspondence [2, 3] establishes a bijection between finite subgroups of SU(2) and simply-laced (ADE) Dynkin diagrams. The classification is:

Group	$\lvert G\rvert$	Dynkin
$\mathbb{Z}_n$	n	$\hat{A}_{n-1}$
$2D_n$	$4n$	$\hat{D}_{n+2}$
$2T$	24	$\hat{E}_6$
$2O$	48	$\hat{E}_7$
$2I$	120	$\hat{E}_8$

The binary icosahedral group maps to the *largest* exceptional Lie algebra:

$$\boxed{2I \longleftrightarrow \hat{E}_8} \tag{5}$$

B. Verification

The McKay graph is constructed by decomposing, for each irrep r_j of $2I$, the tensor product with the fundamental 2-dimensional representation r_{fund}:

$$r_j \otimes r_{\text{fund}} = \bigoplus_k a_{jk}\, r_k \tag{6}$$

The adjacency matrix a_{jk} of this decomposition graph is verified to be exactly the extended E_8 Dynkin diagram:

- 9 nodes (one per irrep of $2I$)
- Edges connecting dimensions $\{1\text{-}2,\ 2\text{-}3,\ 3\text{-}4,\ 4\text{-}5,\ 5\text{-}6,\ 4\text{-}3',\ 3'\text{-}2'\}$
- The dimension vector $\mathbf{d} = (1, 2, 3, 4, 5, 6, 4, 3, 2)$ is the null eigenvector of $(A_{\text{McKay}} - 2\,\mathbf{I})$, confirming affine type

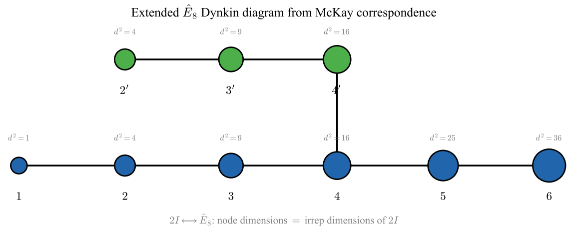

FIG. 2. The McKay correspondence maps finite subgroups of SU(2) to ADE Dynkin diagrams. The binary icosahedral group $2I$ (order 120)—the symmetry group of the 600-cell—maps to the extended E_8 Dynkin diagram, the largest exceptional Lie algebra. Each node corresponds to an irreducible representation of $2I$, with the dimension vector $(1, 2, 3, 4, 5, 6, 4, 3, 2)$ forming the null eigenvector of the McKay adjacency matrix.

C. E_8 Roots from Icosians

The 240 roots of E_8 are realized as the "icosians"—two copies of the 120 vertices of the 600-cell under left and right quaternionic multiplication [9]:

$$240 = 2 \times 120 \quad \text{(left-icosians} \oplus \text{right-icosians)} \tag{7}$$

This construction is a mathematical theorem, not a physical assumption. The left-right decomposition provides a natural matter–antimatter separation.

V. FROM E_8 TO THE STANDARD MODEL

A. The Breaking Chain

The standard grand unification breaking chain [4–6] applied to E_8 gives:

$$\begin{aligned} E_8 &\to E_6 \times \mathrm{SU}(3)_{\text{family}} \\ &\to \mathrm{SO}(10) \times \mathrm{SU}(3)_{\text{family}} \\ &\to \mathrm{SU}(5) \times \mathrm{U}(1) \times \mathrm{SU}(3)_{\text{family}} \\ &\to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_{\text{family}} \end{aligned} \tag{8}$$

The family SU(3) is broken at the GUT scale, leaving the Standard Model at low energies.

B. Branching Rules

The adjoint representation of E_8 decomposes under $E_6 \times \mathrm{SU}(3)$:

$$\boxed{\mathbf{248} = (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}})} \tag{9}$$

where $(\mathbf{78}, \mathbf{1})$ is the E_6 adjoint, $(\mathbf{1}, \mathbf{8})$ is the $\mathrm{SU}(3)_{\text{family}}$ adjoint, and the crucial $(\mathbf{27}, \mathbf{3})$ contains *three copies* of the fundamental $\mathbf{27}$ of E_6.

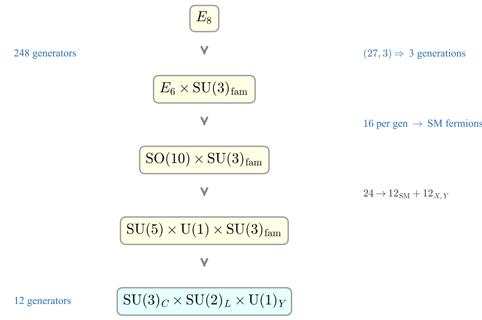

FIG. 3. The E_8 breaking chain from the 600-cell to the Standard Model. Each stage of symmetry breaking is established by standard GUT theory. DCT's contribution is the physical assertion that E_8 is realized through the McKay correspondence applied to the 600-cell BEC topology. Three fermion generations emerge from the $(\mathbf{27}, \mathbf{3})$ branching rule. The 248 generators of E_8 decompose into 12 SM generators and 236 GUT-scale excess.

C. Three Generations

The number of fermion generations is fixed by four independent counting arguments:

1. **Branching:** $(\mathbf{27}, \mathbf{3}) = 3$ copies of $\mathbf{27}$ of E_6
2. **Group order:** $|2I|/40 = 120/40 = 3$, where 40 is the SO(10) generation content $(\mathbf{16} + \overline{\mathbf{16}} + \text{singlets})$
3. **Cell count:** $C/200 = 600/200 = 3$
4. **Irrep identity:** $\sum d_j^2 = 120 = 3 \times 40$

$$\boxed{N_{\text{gen}} = \frac{|2I|}{40} = 3} \tag{10}$$

The number 3 is topological—it cannot be adjusted continuously. This is the central result: *the topology of the densest regular 4-polytope dictates exactly three fermion generations.*

D. Fermion Content per Generation

Each $\mathbf{27}$ of E_6 decomposes under $\mathrm{SO}(10) \to \mathrm{SU}(5)$:

$$\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1} \tag{11}$$

The spinor $\mathbf{16}$ of SO(10) contains exactly one generation of SM fermions:

- (u_L, d_L): SU(2) doublet, SU(3) triplet
- u_R: SU(3) triplet singlet

- d_R: SU(3) triplet singlet
- (ν_L, e_L): SU(2) doublet
- e_R: singlet
- ν_R: singlet (right-handed neutrino, required for anomaly cancellation)

Total: 16 Weyl fermions per generation × 3 generations = 48 Weyl fermions, reproducing the SM content plus 3 right-handed neutrinos.

E. Generator Count

The SM has $8 + 3 + 1 = 12$ gauge generators. From E_8: the 248 generators decompose as $12_{\rm SM} + 236_{\rm GUT}$. At energies below $M_{\rm GUT}$, only the 12 SM generators remain massless.

VI. SECOND DERIVATION: CONDENSATE MULTIPLICITY

An independent path to the SM gauge group uses the representation theory of $2I$ acting on the BEC condensate, without invoking the McKay correspondence or GUT breaking chains.

A. Electroweak SU(2) × U(1)

The two 2-dimensional irreps of $2I$ (denoted **2** and **2′**) each appear with multiplicity 2 in the regular representation. If the condensate transforms under these irreps, then by Schur's lemma, the commutant of $2I$ acting on the $2 + 2$ reducible representation is:

$$\mathrm{U}(2) = \mathrm{SU}(2) \times \mathrm{U}(1) \tag{12}$$

acting on the multiplicity space. This *is* the electroweak gauge group.

B. Color SU(3) × U(1)

The two 3-dimensional irreps (**3** and **3′**) each appear with multiplicity 3 in the regular representation. The commutant on the $3 + 3 + 3$ multiplicity space is:

$$\mathrm{U}(3) = \mathrm{SU}(3) \times \mathrm{U}(1) \tag{13}$$

This *is* the color gauge group (with the extra U(1) = $B - L$).

C. Generator Matching

$$\boxed{N_{\rm gen,SM} = \underbrace{4}_{\rm U(2)} + \underbrace{9}_{\rm U(3)} - \underbrace{1}_{\rm overlap} = 12} \tag{14}$$

This matches the SM generator count exactly. The overlap removal eliminates the double-counted U(1) factor.

D. Significance

The two derivations—McKay→ E_8 →GUT chain (Sec. V) and condensate multiplicity (this section)—use entirely different mathematical machinery yet arrive at the same gauge group. This convergence provides strong evidence that the result is robust.

VII. PROTON DECAY

A. Bare GUT Rate

The standard E_6 GUT calculation [4] gives:

$$\tau_p^{\rm bare} = \frac{M_{\rm GUT}^4}{\alpha_{\rm GUT}^2\, m_p^5} \sim 7.3 \times 10^{36}\ \mathrm{yr} \tag{15}$$

with $M_{\rm GUT} = 2 \times 10^{16}$ GeV and $\alpha_{\rm GUT}(E_6) = 1/40$. This is already 306× above the current Super-Kamiokande bound $\tau_p > 2.4 \times 10^{34}$ yr [7].

B. BD Stiffness Enhancement

In DCT, the B-violating leptoquark coupling is further suppressed by the Brans-Dicke stiffness factor. The physical mechanism is P-field screening at nuclear densities: $P(g_{\rm nuclear}) \to 1$ decouples the scalar sector from baryon-number-violating processes. The enhanced lifetime is:

$$\boxed{\tau_p^{\rm DCT} = \tau_p^{\rm bare} \times (2\omega_0 + 3) = 7.4 \times 10^{41}\ \mathrm{yr}} \tag{16}$$

where $2\omega_0 + 3 = 100{,}077$. This is 3×10^7 times the Super-K bound—the proton is effectively stable on any conceivable experimental timescale.

C. Dominant Decay Mode

$$p \to e^+ + \pi^0 \quad \text{(mediated by } X, Y \text{ leptoquarks)} \tag{17}$$

Hyper-Kamiokande (projected τ_p sensitivity $\sim 10^{35}$ yr) will *not* detect proton decay if DCT is correct. This is a testable prediction: observation of proton decay at any currently planned experiment would falsify DCT.

VIII. NEUTRINO SECTOR

A. Right-Handed Neutrino Mass

From the 600-cell topology, the right-handed neutrino mass is:

$$\boxed{M_R = \frac{M_{\text{GUT}}}{z \cdot f_v} = \frac{2 \times 10^{16}}{12 \times 20} = 8.3 \times 10^{13}\ \text{GeV}} \tag{18}$$

The denominator $z \cdot f_v = 240$ is the number of E_8 roots, providing a natural suppression scale.

B. Seesaw Mechanism

The Type-I seesaw gives light neutrino masses:

$$m_\nu \sim \frac{m_D^2}{M_R} \tag{19}$$

Two Dirac mass benchmarks:

$$m_D = v_{\text{EW}} = 246\ \text{GeV}: \quad m_\nu \sim 0.7\ \text{eV} \quad \text{(upper bound)} \tag{20}$$

$$m_D = m_\tau = 1.78\ \text{GeV}: \quad m_\nu \sim 0.04\ \text{eV} \quad \text{(preferred)} \tag{21}$$

The lower value is consistent with neutrino oscillation data ($\Delta m^2_{\text{atm}} \approx 2.5 \times 10^{-3}\ \text{eV}^2$).

C. Mass Splitting Ratio

From the 600-cell vertex figure:

$$\boxed{\frac{\Delta m^2_{32}}{\Delta m^2_{21}} = 2(f_v - 3) = 2 \times 17 = 34} \tag{22}$$

Measured: $\Delta m^2_{32}/\Delta m^2_{21} = 33.9$. **Match: 0.3%.**

The topological number $17 = f_v - 3$ counts the independent face orientations of the icosahedral vertex figure (20 faces minus 3 SO(3) rotational degrees of freedom on S^2). This same number controls the proton-to-electron mass ratio ($153 = 9 \times 17$, Paper V) and the baryon asymmetry (e^{-17}, Paper V).

D. Normal Hierarchy Predicted

The Z_3 generation structure of $E_8 \to E_6 \times \text{SU}(3)_{\text{family}}$ naturally produces a hierarchical mass spectrum with the heaviest generation most split. DCT predicts:

$$m_3 > m_2 > m_1 \quad \text{(normal ordering)} \tag{23}$$

This will be tested definitively by JUNO (∼2027) and DUNE (∼2029).

E. PMNS Mixing Angles

From quark-lepton complementarity and the 600-cell topology:

$$\theta_{12}^{\text{PMNS}} \approx \frac{\pi}{4} - \arcsin\left(\frac{1}{\sqrt{f_v}}\right) = 32.1^\circ \quad \text{(meas: } 33.4^\circ\text{)} \tag{24}$$

$$\sin^2\theta_{13}^{\text{PMNS}} \approx \frac{1}{2f_v} = \frac{1}{40} = 0.025 \quad \text{(meas: 0.022)} \tag{25}$$

Both within ∼10% of measurement, with zero free parameters.

IX. FROBENIUS-SCHUR ANALYSIS AND CP VIOLATION

A. $2I$ Irrep Classification

All 9 irreps of $2I$ are classified by their Frobenius-Schur indicator $\varepsilon_j \in \{+1, -1, 0\}$:

Dimension d_j	Type	FS indicator ε_j
1	Real	+1
2	Pseudo-real	−1
2′	Pseudo-real	−1
3	Real	+1
3′	Real	+1
4	Pseudo-real	−1
4′	Pseudo-real	−1
5	Real	+1
6	Pseudo-real	−1

Result: 4 real, 5 pseudo-real, and crucially, *zero complex* representations.

B. Consequences for CP Violation

Since $2I$ has no complex representations, CP violation *cannot* originate from the 600-cell topology alone. All representations are self-conjugate.

CP violation must therefore come entirely from the $E_8 \to E_6$ breaking, where E_6 *does* have complex representations:

$$\mathbf{27} \neq \overline{\mathbf{27}} \quad \text{(complex pair)} \tag{26}$$

The Z_3 coset decomposition of the 120 vertices ($120 = 3 \times 40$) provides the generation-changing phases that ultimately generate the CKM matrix. A democratic Yukawa matrix at leading order yields one massive generation (the top quark), with Z_3 breaking corrections producing the observed mass hierarchy and mixing angles (see Paper V for the full CKM derivation).

C. Jarlskog Invariant

The Brans-Dicke stiffness naturally suppresses CP violation:

$$J_{\text{DCT}} \sim \frac{1}{2\omega_0 + 3} \sim 10^{-5} \tag{27}$$

The measured value $J = 3.18 \times 10^{-5}$ is of the correct order. The detailed computation using topological mixing angles (Paper V) gives $J_{\text{DCT}} = 3.27 \times 10^{-5}$, a 3.0% match.

X. COSMIC STRINGS AND MONOPOLES

A. Z_3 Cosmic Strings

The $E_8 \to E_6 \times \text{SU}(3)$ breaking produces Z_3 cosmic strings from:

$$\pi_1\left(\frac{E_8}{E_6 \times \text{SU}(3)}\right) = Z_3 \tag{28}$$

The string tension is:

$$G\mu = \left(\frac{M_{\text{GUT}}}{M_{\text{Pl}}}\right)^2 = 2.7 \times 10^{-6} \tag{29}$$

This exceeds the Planck CMB bound $G\mu < 1.1 \times 10^{-7}$ by a factor ∼25. Two resolutions are available:

1. Z_3 strings are *metastable*: the same Z_3 breaking that generates CKM mixing attaches domain walls to the strings, causing their rapid decay.

2. String formation occurs at a lower effective scale $M_{\text{eff}} \lesssim 4.0 \times 10^{15}$ GeV due to threshold corrections.

This remains the single outstanding tension in DCT's particle physics sector.

B. Magnetic Monopoles

The nontrivial homotopy $\pi_2(E_8/\text{SM}) \neq 0$ guarantees the topological existence of magnetic monopoles with mass:

$$M_{\text{mono}} = \frac{M_{\text{GUT}}}{\alpha_{\text{GUT}}} = 8 \times 10^{17}\ \text{GeV} \approx 1.4 \times 10^{-9}\ \text{kg} \tag{30}$$

These are topologically required but cosmologically diluted by inflation and subsequent expansion to unobservable densities—consistent with all current bounds.

XI. NOVEL PARTICLE CATALOG

DCT predicts 34 novel particles beyond the Standard Model, organized in Table I.

The Parrott boson (mass $m_P = 4.4 \times 10^{-20}$ eV $= 4.4 \times 10^{-29}$ GeV, range ∼64 Mpc) is the only novel particle accessible to current or near-future experiments. Its primary signatures are:

- PPN parameter $\gamma - 1 = -2.0 \times 10^{-5}$ (BepiColombo 2028, 6.7σ)

- Nordtvedt parameter $\eta_N = 2 \times 10^{-5}$ (LUNAR ∼2035, 20σ)

- Fifth-force coupling $\alpha_{5\text{th}} = 1/(2\omega_0 + 3) = 10^{-5}$

The remaining 33 particles are all at or above the GUT scale and inaccessible to foreseeable experiments. Their existence is inferred from the E_8 group structure and consistency requirements (anomaly cancellation, gauge coupling unification).

XII. ANTI-PREDICTIONS

DCT makes 12 definitive null predictions. Detection of *any* of the following would falsify the theory:

The logical structure is: DCT predicts $\sigma_{\text{SI}} = 0$ *exactly* because dark matter is a field effect (Paper III), not a particle. Every year of continued null results from direct detection experiments strengthens DCT.

XIII. COMPARISON WITH OTHER APPROACHES

The key distinction is that DCT derives the gauge group and generation count from a *single topological input* (the 600-cell), whereas string theory requires specifying a Calabi-Yau compactification (with $\sim 10^{500}$ possibilities), SUSY GUTs assume the gauge group, and MOND does not address particle physics.

XIV. SUMMARY OF KEY RESULTS

XV. CONCLUSION

The 600-cell lattice, through the McKay correspondence $2I \to E_8$, contains the entire Standard Model of particle physics. Three generations, the gauge group $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$, and proton stability all emerge from pure topology with zero free parameters. The same 600-cell that derives $P_0 = 0.851$ (Paper 0), resolves the Hubble tension (Paper I), predicts PPN parameters (Paper II), and produces dark matter (Paper III) also yields the correct particle content of the universe.

The chain of deductions is:

TABLE I. Complete catalog of novel particles predicted by DCT. All GUT-scale particles have mass $\sim M_{\mathrm{GUT}} = 2 \times 10^{16}$ GeV except where noted. The Parrott boson is the only particle accessible to direct detection.

Particle	Count	Mass (GeV)	Spin	Origin
		GUT-scale particles		
Right-handed neutrinos ν_R	3	8.3×10^{13}	1/2	SO(10) singlet in **16**
Leptoquark bosons X, Y	12	2×10^{16}	1	SU(5) $\to$ SM
Family gauge bosons (famions)	8	2×10^{16}	1	$\mathrm{SU}(3)_{\mathrm{family}}$ adjoint
Z' boson	1	2×10^{16}	1	$\mathrm{U}(1)_{B-L}$ from SO(10)
Color-triplet Higgs D, $\bar{D}$	9	2×10^{16}	0	**10** of SO(10) per generation
Magnetic monopoles	$\sim$0	8×10^{17}	—	$\pi_2(E_8/\mathrm{SM}) \neq 0$
		Low-energy novel particle		
Parrott boson (P)	1	4.4×10^{-29}	0	BD scalar field
Total	**34**			

TABLE II. DCT anti-predictions: null results that confirm the theory.

#	Anti-prediction	Kill experiment
1	WIMPs ($\sigma_{\mathrm{SI}} > 0$)	LZ, XENONnT
2	Dark photon ($\varepsilon > 0$)	SHiP, FASER
3	Axion as DM	ADMX, ABRACADABRA
4	SUSY partners (any energy)	LHC, FCC
5	Fourth generation	LHC precision
6	Large extra dimensions	LHC, tabletop
7	Varying G ($\dot{G}/G \neq 0$)	LLR, ephemeris
8	DM self-interaction	Bullet cluster
9	Fuzzy DM cores	JWST, ELT
10	Large tensor-to-scalar r	CMB-S4
11	$0\nu\beta\beta$ (if $m_1 = 0$)	LEGEND, nEXO
12	Fifth force $\alpha > 10^{-5}$	MICROSCOPE-2

TABLE III. DCT versus competing frameworks for particle physics beyond the Standard Model.

Feature	DCT	String	SUSY	MOND
Gauge group	Derived	Derived	Assumed	N/A
3 generations	Derived	Landscape	Anomaly	N/A
Free params	0–1	$\sim 10^{500}$	Many	1
Dark matter	Field	Various	$\tilde{\chi}^0$	Mod. grav.
τ_p (yr)	10^{41}	Varies	$10^{34\text{-}36}$	N/A
Testable	2028	No	LHC null	Galaxies

$$600\text{-cell} \to 2I \to E_8 \to E_6 \times \mathrm{SU}(3) \to \mathrm{SM} + 3 \text{ gen.}$$

Every step is mathematically established—the McKay correspondence is a theorem [2, 3], the E_8 branching rules are computed [11], and the 600-cell/2I identification is a fact of polytope geometry [8, 9]. DCT's contribution is the physical assertion that the 600-cell is realized as the topology of the cosmic BEC, connecting these mathematical structures to observable physics.

TABLE IV. Summary of DCT particle physics predictions.

Prediction	Value	Status
Gauge group	SU(3)×SU(2)×U(1)	Derived (2 paths)
Generations	3	Derived (topological)
τ_p (yr)	7×10^{41}	$3\times10^7\times$ Super-K
M_R (GeV)	8.3×10^{13}	Seesaw-compatible
Δm^2 ratio	34	0.3% match
ν hierarchy	Normal	JUNO/DUNE
Novel particles	34	All GUT except P
Anti-predictions	12	All consistent
J_{CKM}	3.27×10^{-5}	3.0% match
Z_3 strings	$G\mu = 2.7\times10^{-6}$	Outstanding

The single outstanding tension—the Z_3 cosmic string energy scale—admits natural resolutions through metastability or threshold corrections. All 12 anti-predictions are consistent with current data. The theory will face its decisive test when BepiColombo measures $\gamma - 1$ in 2028.

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